IIB solutions with N>28 Killing spinors are maximally supersymmetric

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Abstract: We show that all IIB supergravity backgrounds which admit more than 28 Killing spinors are maximally supersymmetric. In particular, we find that for all $N > 28$ backgrounds the supercovariant curvature vanishes, and that the quotients of maximally supersymmetric backgrounds either preserve all 32 or $N < 29$ supersymmetries.

Keywords: Superstring Vacua, Extended Supersymmetry, Supergravity Models.

Contents

1. Introduction

Recently, it has been realized that there are restrictions on the existence of type II and eleven-dimensional supergravity backgrounds with near maximal number of supersymmetries. This was initiated in [1] where it was shown that IIB backgrounds with $N = 31$ supersymmetries are maximally supersymmetric. Later this was extended to IIA backgrounds in [2]. These results mostly follow from an analysis of the algebraic Killing spinor equations.

Eleven-dimensional supergravity backgrounds with 31 supersymmetries also admit an additional Killing spinor and so are maximally supersymmetric. To show this, one first proves that the supercovariant curvature of $N = 31$ backgrounds vanishes subject to the field equations and Bianchi identities of eleven-dimensional supergravity [3]. This demonstrates that the $N = 31$ backgrounds are locally maximally supersymmetric. Then one shows that there are no discrete quotients of maximally supersymmetric backgrounds which preserve 31 supersymmetries [4]. These results exclude the existence of preonic backgrounds [5] in type II and eleven-dimensional supergravities.

Most of the above results have been obtained by adapting the spinorial geometry technique for solving Killing spinor equations [6] to backgrounds with near maximal number of supersymmetries. The investigation of discrete quotients of maximally supersymmetric backgrounds relies on techniques developed in [7, 8]. Similar results hold for some supergravities in lower dimensions [9]. However in non-maximal supergravities in four and five dimensions, it is possible to construct preonic backgrounds as discrete quotients of maximally supersymmetric ones [10].

In this paper, we show that IIB backgrounds with $N > 28$ supersymmetries are maximally supersymmetric. For this, we first use the property that $N > 24$ supersymmetric IIB backgrounds are homogeneous spaces [11]. This in particular implies that the one-form field strength P vanishes, $P = 0$. As a result the algebraic Killing spinor equation of IIB supergravity is linear over the complex numbers and so it always has an even number of solutions. In addition, an application of the spinorial geometry technique reveals that if $N = 30$, then the three-form field strength vanishes as well, $G = 0$. Therefore one concludes that for all $N > 28$ IIB backgrounds, the algebraic Killing spinor equation implies $P = G = 0$.

This in turn implies that the gravitino Killing spinor equation also has even number of solutions [1]. Therefore to prove our result, we should exclude the existence of IIB backgrounds with 30 supersymmetries. For this we explore the integrability conditions of the gravitino Killing spinor equation. The analysis is similar in spirit as that for the $N = 31$ backgrounds of eleven-dimensional supergravity [3]. In particular, we show that the curvature $\mathcal{R}$ of the supercovariant connection vanishes, $\mathcal{R} = 0$, subject to the Bianchi identities and field equations of IIB supergravity. This demonstrates that $N > 28$ IIB backgrounds are locally maximally supersymmetric. Using the classification of maximally supersymmetric IIB backgrounds [12], one concludes that the $N > 28$ backgrounds must be locally isometric to one of the following solutions: Minkowski space $\mathbb{R}^{9,1}$, the Freund-Rubin space $AdS_5 \times S^5$ [13] and the maximally supersymmetric plane wave [14].

Finally, we show that one cannot construct $28 < N < 32$ IIB backgrounds as discrete quotients of the maximally supersymmetric ones. To establish our result, we lift the generators of the discrete symmetry group to $Spin_c(9,1) = \mathrm{Spin}(9,1) \times_{\mathbb{Z}_2} \mathrm{U}(1)$ and prove that there are no invariant spinors that span a 30-dimensional subspace. This computation relies on the lift of the generators of the discrete group to the $\mathrm{Spin}(9,1)$ group investigated in [7, 8]. Our lift has an additional phase along the $\mathrm{U}(1)$ direction of $Spin_c(9,1)$. Our final result is in agreement with a conjecture in [15] which was formulated using the assumption that the Killing spinors transform under certain representations of subgroups of $\mathrm{Spin}(9,1)_c$.

This paper is organized as follows. In section two, we show using the algebraic Killing spinor equation that for $N > 28$ supersymmetric IIB backgrounds the three-form field strength vanishes, $G = 0$. In section three, we describe the conditions that the field equations and the Bianchi identities impose on the holonomy of the supercovariant IIB connection. In sections four, five and six, we demonstrate that the supercovariant curvature of all $N > 28$ IIB backgrounds vanishes. In section seven, we exclude the possibility of constructing $28 < N < 32$ backgrounds as discrete quotients of Minkowski space $\mathbb{R}^{9,1}$, $AdS_5 \times S^5$ and the maximally supersymmetric plane wave, and in section eight we give our conclusions.

2. Algebraic Killing spinor equation

The algebraic Killing spinor equation (KSE) of IIB supergravity [16, 13, 17] is

$$P_A \Gamma^A C \epsilon^* + \frac{1}{24} G_{ABC} \Gamma^{ABC} \epsilon = 0 \,, \tag{2.1}$$

where P and G are the (complex) one- and three-form field strengths, respectively, C is the charge conjugation matrix, and ϵ is a complex Weyl $Spin_c(9,1)$ spinor. For our spinor conventions, see e.g. [18]. It is known that IIB backgrounds with more than 24 supersymmetries are locally homogeneous [11]. In particular, this implies that the scalars are constant and hence that their field strength vanishes, $P = 0$. The vanishing of P has the important implication that the dilatino KSE becomes linear over the complex numbers. In other words, it has an even number of solutions which can be expressed as $(\epsilon^r, i\epsilon^r)$ pairs.

The aim is to show that the algebraic Killing spinor equation for $N > 28$ backgrounds implies $G = 0$. It is known that if $N = 32$, the algebraic Killing spinor equation implies that $P = G = 0$ [12]. So it remains to prove the statement for $N = 30$. Since the algebraic Killing spinor equation for $P = 0$ is linear over the complex numbers, the solution spans a complex hyperplane in the space of spinors at every spacetime point. So it has a normal ν with respect to the standard Majorana inner product. Using spinorial geometry and in particular the gauge symmetry of the Killing spinor equations, the normal direction ν can be chosen of the form [1]

$$\begin{aligned} \text{Spin}(7) \ltimes \mathbb{R}^8 : \quad & \nu = (n + im)(e_5 + e_{12345}) \,, \\ \text{SU}(4) \ltimes \mathbb{R}^8 : \quad & \nu = (n - \ell + im)e_5 + (n + \ell + im)e_{12345} \,, \\ G_2 : \quad & \nu = n(e_5 + e_{12345}) + im(e_1 + e_{234}) \,, \end{aligned} \tag{2.2}$$

corresponding to the three different orbits of $\text{Spin}(9,1)$ in the space of negative chirality Weyl spinors [18], where n, m and ℓ are real spacetime functions. Choosing the solutions orthogonal to the above normals, they can be expressed as

$$\epsilon^r = \sum_{s=1}^{15} z^r{}_s \eta^s \,, \tag{2.3}$$

where η_i is a basis normal to ν and z is an invertible 15×15 matrix of spacetime dependent complex functions, see [19] for more details. Consequently, the Killing spinor equation

becomes

$$G_{ABC}\Gamma^{ABC}\eta^r = 0 \ . \tag{2.4}$$

Since in all three cases the normal ν can be arranged to point only in at most three different directions $e_5 + e_{12345}, i(e_5 - e_{12345})$ and $(e_1 + e_{234})$, the bases (η^s) can be chosen such that they contain 13 common elements. The other two elements depend on the choice of orbit and have to be considered case by case. We will first analyze the constraints obtained from (2.4) acting on the 13 common elements, and afterwards specialize to the three different cases.

The 13 common basis elements η^r, $r = 1, \ldots, 13$, are given by those of the 16 basis elements of the Majorana-Weyl representation of $\mathrm{Spin}(9,1)$ which are linearly independent from $1 + e_{1234}, i(1 - e_{1234})$ and $(e_{15} + e_{2345})$. Substituting this into the algebraic Killing spinor equation (2.4), we find that the non-vanishing components of G satisfy

$$G_{m\bar{1}\bar{m}} = -\frac{1}{2}G_{\bar{2}\bar{3}\bar{4}}, \quad G_{-+\bar{1}} = \frac{1}{2}G_{\bar{2}\bar{3}\bar{4}}, \quad G_{+1\bar{1}} = G_{+m\bar{m}},$$
$$G_{1m\bar{m}} = -\frac{1}{2}G_{234}, \quad G_{-+1} = \frac{1}{2}G_{234}, \tag{2.5}$$

where $m = 2, 3, 4$, and there is no summation in the repeated m indices. Hence there are only three independent non-vanishing components left of the original 120.

Now the analysis splits up for the three different orbits, since the two additional basis elements η^r, $r = 14, 15$, differ:

- The simplest orbit is $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$, in which case the two additional basis elements are $\eta^{14} = 1 - e_{1234}$ and $\eta^{15} = e_{15} + e_{2345}$. When inserted into the dilatino variation, the former implies $G_{+1\bar{1}} = 0$ and the latter implies $G_{234} = G_{\bar{2}\bar{3}\bar{4}} = 0$. Hence $G = 0$ in this case.

- In the $\mathrm{SU}(4) \ltimes \mathbb{R}^8$ case, one has $\eta^{14} = e_{15} + e_{2345}$. This leads to $G_{234} = G_{\bar{2}\bar{3}\bar{4}} = 0$. The remaining basis element is given by $\eta^{15} = (n - \ell + im)1 - (n + \ell + im)e_{1234}$ and implies $G_{+1\bar{1}} = 0$. Hence G also vanishes for the $\mathrm{SU}(4) \ltimes \mathbb{R}^8$ orbit.

- The remaining case is the G_2 orbit. For this, $\eta^{14} = 1 - e_{1234}$, which leads to the vanishing of $G_{+1\bar{1}}$. The other two components of G are set to zero by $\eta^{15} = m(1 + e_{1234}) + in(e_{15} + e_{2345})$. Hence $G = 0$ for this orbit as well.

Therefore we conclude that for $N > 28$ IIB backgrounds, $P = G = 0$ as a consequence of the homogeneity and the algebraic Killing spinor equation. As we have mentioned, if $G = 0$, the gravitino Killing spinor equation has an even number of solutions. Thus $N > 28$ IIB backgrounds can have either 30 or 32 supersymmetries. We shall exclude the existence of $N = 30$ backgrounds by investigating the gravitino Killing spinor equation.

3. Supercovariant curvature and holonomy

3.1 Supercurvature

Assuming $G = 0$, the curvature $\mathcal{R} = [\mathcal{D}, \mathcal{D}]$ of the covariant connection $\mathcal{D}$ of IIB supergravity can be expanded [12] as

$$\mathcal{R}_{MN} = \text{Re}\,\mathcal{R}_{MN} + i\,\text{Im}\,\mathcal{R}_{MN} = \frac{1}{2}(T_{MN}^2)_{PQ}\Gamma^{PQ} + \frac{1}{4!}(\hat{T}_{MN}^4 + i\tilde{T}_{MN}^4)_{Q_1...Q_4}\Gamma^{Q_1...Q_4}\,, \quad (3.1)$$

where

$$
\begin{aligned}
(T_{MN}^2)_{P_1P_2} &= \frac{1}{4}R_{MN,P_1P_2} - \frac{1}{12}F_{M[P_1}{}^{Q_1Q_2Q_3}F_{|N|P_2]Q_1Q_2Q_3}\,,\\
(T_{MN}^4)_{P_1...P_4} &= \frac{i}{2}D_{[M}F_{N]P_1...P_4} + \frac{1}{2}F_{MNQ_1Q_2[P_1}F_{P_2P_3P_4]}{}^{Q_1Q_2}\,,
\end{aligned}
\quad (3.2)
$$

and R is the Riemann curvature, F is the self-dual five-form field strength and $T^4 = \hat{T}^4 + i\tilde{T}^4$. Observe that $\tilde{T}^4$ contains only the covariant derivative of F. We have made use of the self-duality of F to simplify these expressions. The components of T^2 and T^4 are not all independent but are restricted by the Bianchi identities of R and F, $(dF = 0)$, and the field equations of IIB supergravity. In particular, using the expressions of T^2 and T^4 in terms of the physical fields (3.2) and the Bianchi identities, one finds that

$$
\begin{aligned}
(T_{MN}^2)_{P_1P_2} &= (T_{P_1P_2}^2)_{MN}\,,\\
(T_{M[P_1}^2)_{P_2P_3]} &= 0\,,\\
(T_{[P_1P_2}^4)_{P_3P_4P_5P_6]} &= 0\,.
\end{aligned}
\quad (3.3)
$$

Next observe that $\Gamma^N\mathcal{R}_{MN}$ is a linear combination of the field equations [19]. Making use of this and of (3.3), we find

$$
\begin{aligned}
(T_{MN}^2)_P{}^N &= 0\,,\\
(T_{MN}^4)_{P_1P_2P_3}{}^N &= 0\,,\\
(T_{M[P_1}^4)_{P_2P_3P_4P_5]} &= -\frac{1}{5!}\epsilon_{P_1P_2P_3P_4P_5}{}^{Q_1Q_2Q_3Q_4Q_5}(T_{M[Q_1}^4)_{Q_2Q_3Q_4Q_5]}\,.
\end{aligned}
\quad (3.4)
$$

Also note that $(T^4{}_{P_1(M})_{N)P_2P_3P_4}$ is totally antisymmetric in P_1, P_2, P_3, P_4.

One of the consequences of the first condition in (3.4), or equivalently from the Einstein field equation and $P = G = 0$, is that the scalar curvature of the spacetime vanishes, i.e. $R = 0$. Furthermore, on imposing the Einstein equations, and using the self-duality of F, it is straightforward to show that $(T_{MN}^2)_{PQ} = \frac{1}{4}W_{MNPQ}$, where W is the spacetime Weyl tensor. The expressions in this subsection do not rely on the existence of Killing spinors and are therefore valid for all backgrounds.

3.2 Holonomy

It is clear from the expression for $\mathcal{R}$ in the previous section that the (reduced) holonomy of the supercovariant connection of IIB backgrounds with $P = G = 0$ is contained in $\text{SL}(16, \mathbb{C})$. This is a subgroup of $\text{SL}(32, \mathbb{R})$ which is the holonomy of the supercovariant

connection for generic IIB backgrounds [20]. It immediately follows from the integrability conditions of the gravitino Killing spinor equation and in particular of

$$\mathcal{R}\epsilon^r = 0 \tag{3.5}$$

that the holonomy of a spacetime with $N = 2n$ supersymmetries reduces to a subgroup of $\mathrm{SL}(16 - n, \mathbb{C}) \ltimes \oplus_n \mathbb{C}^{16-n}$. Therefore on the grounds of holonomy, one expects that there are supersymmetric $P = G = 0$ backgrounds with any even number $N \le 32$ of supersymmetries. However as we shall show the $N = 30$ case will be excluded.

Let $(\epsilon^r, \tilde{\epsilon}^p)$ be a complex (local) basis in the space of spinors where ϵ^r, $r = 1, \ldots, n$ is a basis in the space of Killing spinors, $N = 2n$, and $p = n + 1, \ldots, 16$. Moreover, let ν^q, $q = 1, \ldots 16 - n$, be a basis in the space normal to the Killing spinors with respect to the Majorana inner product B. Using a similar argument to the one we have employed for M-theory [3], the supercurvature of a spacetime with $N = 2n$ Killing spinors can be locally written as

$$\mathcal{R}_{MN,ab'} = U_{MN,rq}\epsilon^r_a \nu^q_{b'} + U_{MN,pq}\tilde{\epsilon}^p_a \nu^q_{b'} , \tag{3.6}$$

where a, b' are chiral and anti-chiral spinorial indices, respectively, and $U_{MN,rq}$ and $U_{MN,pq}$ are complex spacetime two-forms. Clearly, in writing the supercovariant curvature in this way it automatically satisfies the integrability condition (3.5). Moreover, the above condition can be written in any other basis in the space of spinors. In particular, we may choose say a Majorana or another suitable basis η^r and write

$$\mathcal{R}_{MN,ab'} = u_{MN,rq}\eta^r_a \nu^q_{b'} , \tag{3.7}$$

where again u are complex two-forms on the spacetime. On the other hand we know that

$$\eta_a \theta_{b'} = -\frac{1}{16} \sum_{k=0}^{2} \frac{1}{(2k)!} B(\eta, \Gamma_{A_1 A_2 \ldots A_{2k}}\theta)(\Gamma^{A_1 A_2 \ldots A_{2k}})_{ab'} , \tag{3.8}$$

This in turn gives

$$\mathcal{R}_{MN,A_1 \ldots A_{2k}} = -\frac{1}{16} u_{MN,rq} B(\eta^r, \Gamma_{A_1 A_2 \ldots A_{2k}}\nu^q) . \tag{3.9}$$

The complex spacetime two-forms u are not all independent. One condition arises from the requirement that the holonomy of the supercovariant connection for all backgrounds is a subgroup of $\mathrm{SL}(16, \mathbb{C})$. This in particular gives

$$u_{MN,rq} B(\eta^r, \nu^q) = 0 . \tag{3.10}$$

Taking this into account, the number of independent two forms u for $N = 2n$ supersymmetric backgrounds is equal to the dimension of $\mathrm{SL}(16 - n, \mathbb{C}) \ltimes \oplus_n \mathbb{C}^{16-n}$ as expected. In the cases we shall investigate below, the basis η^r is chosen in such a way that (3.10) is automatically satisfied.

Apart from (3.10), there are additional conditions on the two-forms u. In particular those that arise from the Bianchi identities and field equations of IIB supergravity described in the previous section. These can potentially further reduce the holonomy of the supercovariant connection to a proper subgroup of $\mathrm{SL}(16 - n, \mathbb{C}) \ltimes \oplus_n \mathbb{C}^{16-n}$.

In the special case for which $N = 30$, and so $n = 15$, that we are interested in, there is a unique (complex) normal direction ν to the Killing spinors. The holonomy of the supercovariant connection is contained in $\mathbb{C}^{15}$. Taking into account the condition (3.10), the supercovariant curvature is determined in terms of 15 complex spacetime two-forms u, as expected. Furthermore, we shall show that all these 15 two-forms vanish subject to the Bianchi identities and field equations of IIB supergravity. As a result $\mathcal{R} = 0$ and $N = 30$ IIB supergravity backgrounds are locally maximally supersymmetric. There are three cases to consider depending on the orbit type of the normal to the Killing spinors.

4. Spin(7)-invariant normal

The normal direction can be chosen as $\nu = e_5 + e_{12345}$. A suitable basis such that (3.10) is automatically satisfied is

$$\eta^{\bar{\alpha}\bar{\beta}} = e_{\alpha\beta}, \qquad\qquad \eta^{\bar{\alpha}} = e_{\alpha 5},$$
$$\eta^{\alpha} = \frac{1}{6}\epsilon^{\alpha\beta_1\beta_2\beta_3} e_{\beta_1\beta_2\beta_3 5}, \qquad\qquad \eta^+ = 1 - e_{1234}, \qquad (4.1)$$

where $\alpha, \beta = 1, 2, 3, 4$. By considering the relation

$$(T^2)_{P_1 P_2} = -\frac{1}{16} u_r B(\eta^r, \Gamma_{P_1 P_2} \nu), \qquad (4.2)$$

where the form indices MN have been suppressed in (T^2) and in u_r, we find the relations

$$(T^2)_{+-} = (T^2)_{-\mu} = (T^2)_{-\bar{\mu}} = 0, \qquad (T^2)_{+\mu} = -\frac{1}{8} u_\mu, \qquad (T^2)_{+\bar{\mu}} = -\frac{1}{8} u_{\bar{\mu}},$$
$$(T^2)_{\mu\nu} = -\frac{1}{16} \epsilon_{\mu\nu}{}^{\bar{\beta}_1\bar{\beta}_2} u_{\bar{\beta}_1\bar{\beta}_2}, \qquad (T^2)_{\mu\bar{\nu}} = \frac{1}{8} u_+ \delta_{\mu\bar{\nu}}, \qquad (T^2)_{\bar{\mu}\bar{\nu}} = \frac{1}{8} u_{\bar{\mu}\bar{\nu}}. \qquad (4.3)$$

Note that $u_{MN,r}$ are complex valued. To proceed, observe that

$$u_+ = 2(T^2)_\alpha{}^\alpha \qquad (4.4)$$

and hence, making use of the constraint $(T^2_{MN})_{P_1 P_2} = (T^2_{P_1 P_2})_{MN}$, we find that

$$(T^2_{\alpha\bar{\beta}})_{\mu\bar{\nu}} = \frac{1}{16} (T^2{}_\rho{}^\rho)_\lambda{}^\lambda \delta_{\alpha\bar{\beta}} \delta_{\mu\bar{\nu}}. \qquad (4.5)$$

Next note that (making use of $(T^2)_{-\mu} = 0$)

$$0 = (T^2{}_{N\bar{\beta}})_\mu{}^N = (T^2{}_{\sigma\bar{\beta}})_\mu{}^\sigma + (T^2{}_{\bar{\sigma}\bar{\beta}})_\mu{}^{\bar{\sigma}}. \qquad (4.6)$$

However,

$$(T^2{}_{\bar{\sigma}\bar{\beta}})_\mu{}^{\bar{\sigma}} = -\frac{1}{16} \epsilon_\mu{}^{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} u_{\bar{\beta}_1\bar{\beta},\bar{\beta}_2\bar{\beta}_3} = -\frac{1}{2} \epsilon_\mu{}^{\bar{\beta}_1\bar{\beta}_2\bar{\beta}_3} (T^2{}_{\bar{\beta}_1\bar{\beta}})_{\bar{\beta}_2\bar{\beta}_3} = 0 \qquad (4.7)$$

by the Bianchi identity. Hence, it follows that $(T^2{}_{\sigma\bar\beta})_\mu{}^\sigma = 0$, which implies that $(T^2{}_\rho{}^\rho)_\lambda{}^\lambda = 0$. Hence

$$(T^2{}_{\alpha\bar\beta})_{\mu\bar\nu} = 0 \tag{4.8}$$

so

$$u_{\alpha\bar\beta,+} = 0 \ . \tag{4.9}$$

Similarly, we also have

$$(T^2{}_{+\alpha})_{\mu\bar\nu} = \frac{1}{8}u_{+\alpha,+}\delta_{\mu\bar\nu} \tag{4.10}$$

and hence $u_{+\alpha,+} = 2(T^2{}_{+\alpha})_\lambda{}^\lambda$, so

$$(T^2{}_{+\alpha})_{\mu\bar\nu} = \frac{1}{4}(T^2{}_{+\alpha})_\lambda{}^\lambda\delta_{\mu\bar\nu} \ . \tag{4.11}$$

Next, note that

$$0 = (T^2{}_{N+})_\mu{}^N = (T^2{}_{\sigma+})_\mu{}^\sigma + (T^2{}_{\bar\sigma+})_\mu{}^{\bar\sigma} \ , \tag{4.12}$$

where we have made use of $(T^2)_{+-} = 0$. However, $(T^2{}_{\bar\sigma+})_\mu{}^{\bar\sigma} = 0$ from the Bianchi identity, hence $(T^2{}_{\sigma+})_\mu{}^\sigma = 0$ also. This implies that $(T^2{}_{+\alpha})_\lambda{}^\lambda = 0$, so $(T^2{}_{+\alpha})_{\mu\bar\nu} = 0$. Therefore $u_{+\alpha,+} = 0$. Also, $(T^2{}_{+\alpha})_{\mu\bar\nu} = 0$ implies that $(T^2{}_{+\bar\alpha})_{\mu\bar\nu} = 0$ (as T^2 is real), hence it follows that $u_{+\bar\alpha,+} = 0$.

The vanishing of $(T^2{}_{\mu\bar\nu})_{-\alpha}$, $(T^2{}_{\mu\bar\nu})_{-\bar\alpha}$, and $(T^2{}_{\mu\bar\nu})_{+-}$ also implies that $u_{-\alpha,+} = 0$, $u_{-\bar\alpha,+} = 0$ and $u_{+-,+} = 0$. Next, consider

$$(T^2{}_{\alpha\beta})_{\mu\bar\nu} = \frac{1}{8}u_{\alpha\beta,+}\delta_{\mu\bar\nu} \ . \tag{4.13}$$

Contracting with $\epsilon^{\alpha\beta\mu}{}_{\bar\lambda}$ and using the Bianchi identity we find $u_{\alpha\beta,+} = 0$, so $(T^2{}_{\alpha\beta})_{\mu\bar\nu} = 0$. As T^2 is real, this implies that $(T^2{}_{\bar\alpha\bar\beta})_{\mu\bar\nu} = 0$, which then fixes $u_{\bar\alpha\bar\beta,+} = 0$. So all components of u_+ vanish.

Next, recall that $(T^2)_{+\mu} = -\frac{1}{8}u_\mu$. Then the vanishing of $(T^2{}_{+\mu})_{\alpha\bar\beta}$, $(T^2{}_{+\mu})_{-\alpha}$, $(T^2{}_{+\mu})_{-\bar\alpha}$ and $(T^2{}_{+\mu})_{-+}$ implies that

$$u_{\alpha\bar\beta,\mu} = 0, \qquad u_{-\alpha,\mu} = 0, \qquad u_{-\bar\alpha,\mu} = 0, \qquad u_{-+,\mu} = 0 \ . \tag{4.14}$$

Next note that

$$(T^2{}_{\alpha\beta})_{+\mu} = (T^2{}_{+\mu})_{\alpha\beta} = -\frac{1}{2}\epsilon_{\alpha\beta}{}^{\bar\rho\bar\sigma}(T^2{}_{+\mu})_{\bar\rho\bar\sigma} \ . \tag{4.15}$$

However, we also have $(T^2{}_{+[\mu})_{\bar\rho\bar\sigma]} = 0$. Together with $(T^2)_{\mu\bar\sigma} = 0$ this implies that $(T^2{}_{+\mu})_{\bar\rho\bar\sigma} = 0$ and hence $(T^2{}_{\alpha\beta})_{+\mu} = 0$ also. Hence $u_{\alpha\beta,\mu} = 0$. Furthermore, $(T^2{}_{\bar\rho\bar\sigma})_{+\mu} = 0$ implies that $u_{\bar\alpha\bar\beta,\mu} = 0$ as well.

Next consider $(T^2)_{+\bar\mu} = -\frac{1}{8}u_{\bar\mu}$. The vanishing of $(T^2{}_{+\bar\mu})_{\alpha\bar\beta}$, $(T^2{}_{+\bar\mu})_{-\alpha}$, $(T^2{}_{+\bar\mu})_{-\bar\alpha}$, $(T^2{}_{+\bar\mu})_{-+}$, $(T^2{}_{+\bar\mu})_{\alpha\beta}$ and $(T^2{}_{\bar\alpha\bar\beta})_{+\bar\mu}$ implies that

$$u_{\alpha\bar\beta,\bar\mu} = 0, \quad u_{-\alpha,\bar\mu} = 0, \quad u_{-\bar\alpha,\bar\mu} = 0, \quad u_{-+,\bar\mu} = 0, \quad u_{\alpha\beta,\bar\mu} = 0, \quad u_{\bar\alpha\bar\beta,\bar\mu} = 0 \ . \tag{4.16}$$

Next consider the constraint $(T^2)_{\bar{\mu}\bar{\nu}} = \frac{1}{8}u_{\bar{\mu}\bar{\nu}}$. As

$$(T^2{}_{\alpha\bar{\beta}})_{\bar{\mu}\bar{\nu}} = (T^2{}_{\bar{\mu}\bar{\nu}})_{\alpha\bar{\beta}} = 0 \,, \tag{4.17}$$

it follows that $u_{\alpha\bar{\beta},\bar{\mu}\bar{\nu}} = 0$. Similarly, the vanishing of $(T^2{}_{\bar{\mu}\bar{\nu}})_{-\alpha}$, $(T^2{}_{\bar{\mu}\bar{\nu}})_{-\bar{\alpha}}$, $(T^2{}_{\bar{\mu}\bar{\nu}})_{+-}$, $(T^2{}_{\bar{\mu}\bar{\nu}})_{+\alpha}$ and $(T^2{}_{\bar{\mu}\bar{\nu}})_{+\bar{\alpha}}$ implies that

$$u_{-\alpha,\bar{\mu}\bar{\nu}} = 0 \,, \quad u_{-\bar{\alpha},\bar{\mu}\bar{\nu}} = 0, \quad u_{+-,\bar{\mu}\bar{\nu}} = 0, \quad u_{+\alpha,\bar{\mu}\bar{\nu}} = 0, \quad u_{+\bar{\alpha},\bar{\mu}\bar{\nu}} = 0 \,. \tag{4.18}$$

Next consider the Bianchi identity

$$(T^2{}_{\alpha[\beta})_{\bar{\mu}\bar{\nu}]} = 0 \,. \tag{4.19}$$

As $u_+ = 0$, it follows that $(T^2{}_{\alpha\bar{\nu}})_{\beta\bar{\nu}} = 0$, and hence $(T^2{}_{\alpha\beta})_{\bar{\mu}\bar{\nu}} = 0$. Therefore $u_{\alpha\beta,\bar{\mu}\bar{\nu}} = 0$. Also

$$(T^2_{\bar{\alpha}\bar{\beta}})_{\bar{\mu}\bar{\nu}} = -\frac{1}{2}\epsilon_{\bar{\mu}\bar{\nu}}{}^{\lambda_1\lambda_2}(T^2{}_{\bar{\alpha}\bar{\beta}})_{\lambda_1\lambda_2} = 0 \,, \tag{4.20}$$

so $u_{\bar{\alpha}\bar{\beta},\bar{\mu}\bar{\nu}} = 0$. Hence all components of $u_{\bar{\mu}\bar{\nu}}$ vanish.

To summarize, these constraints fix all components of u_r to vanish, with the exception of $u_{+A,B}$ where A, B are $su(4)$ indices. As

$$(T^2{}_{+A})_{+B} = -\frac{1}{8}u_{+A,B} \,, \tag{4.21}$$

it follows that $u_{+A,B}$ is symmetric in A, B.

Next consider the 4-forms. It turns out that all components of T^4 are forced to vanish by the above constraints with the exception of

$$(T^4)_{+\mu\nu\rho} = -\frac{1}{4}u_{\bar{\alpha}}\epsilon^{\bar{\alpha}}{}_{\mu\nu\rho} \,, \qquad\qquad (T^4)_{+\mu\nu\bar{\rho}} = \frac{1}{8}u_\mu\delta_{\nu\bar{\rho}} - \frac{1}{8}u_\nu\delta_{\mu\bar{\rho}} \,,$$

$$(T^4)_{+\mu\bar{\nu}\bar{\rho}} = -\frac{1}{8}\delta_{\mu\bar{\nu}}u_{\bar{\rho}} + \frac{1}{8}\delta_{\mu\bar{\rho}}u_{\bar{\nu}} \,, \qquad (T^4)_{+\bar{\mu}\bar{\nu}\bar{\rho}} = -\frac{1}{4}u_\alpha\epsilon^{\alpha}{}_{\bar{\mu}\bar{\nu}\bar{\rho}} \,. \tag{4.22}$$

Using (4.21), this implies that

$$(T^4)_{+\mu\nu\rho} = 2(T^2)_{+\bar{\alpha}}\epsilon^{\bar{\alpha}}{}_{\mu\nu\rho} \,, \qquad\qquad (T^4)_{+\mu\nu\bar{\rho}} = (T^2)_{+\nu}\delta_{\mu\bar{\rho}} - (T^2)_{+\mu}\delta_{\nu\bar{\rho}} \,,$$

$$(T^4)_{+\bar{\mu}\bar{\nu}\rho} = (T^2)_{+\bar{\nu}}\delta_{\bar{\mu}\rho} - (T^2)_{+\bar{\mu}}\delta_{\bar{\nu}\rho} \,, \qquad (T^4)_{+\bar{\mu}\bar{\nu}\bar{\rho}} = 2(T^2)_{+\alpha}\epsilon^{\alpha}{}_{\bar{\mu}\bar{\nu}\bar{\rho}} \,. \tag{4.23}$$

This implies that T^4 is entirely real, so that F is covariantly constant. Furthermore, $(T^4{}_{+A_1})_{+A_2A_3A_4}$ is totally antisymmetric in A_1, A_2, A_3, A_4. Recall that $(T^4{}_{M[P_1})_{P_2P_3P_4P_5]}$ is self-dual in the five anti-symmetrized indices. Hence $(T^4{}_{+\alpha_1})_{+\alpha_2\alpha_3\alpha_4}$ must vanish. Then (4.23) implies that $(T^2{}_{+\alpha})_{+\bar{\beta}} = 0$.

Also consider

$$(T^4{}_{+\alpha})_{+\mu\nu\bar{\rho}} = \delta_{\mu\bar{\rho}}(T^2{}_{+\alpha})_{+\nu} - \delta_{\nu\rho}(T^2{}_{+\alpha})_{+\mu} \,. \tag{4.24}$$

Contracting this identity gives

$$(T^4{}_{+\alpha})_{+\mu\lambda}{}^{\lambda} = -3(T^2{}_{+\alpha})_{+\mu} \,. \tag{4.25}$$

However, the self-duality condition implies that $(T^4{}_{+\alpha})_{+\mu\lambda}{}^{\lambda} = 0$, and hence $(T^2{}_{+\alpha})_{+\beta} = 0$ also. Therefore, all components of T^2 and T^4 are constrained to vanish.

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5. SU(4) ⋉ ℝ⁸-invariant normal

The normal spinor direction is taken to be

$$\nu = (n - \ell + im)e_5 + (n + \ell + im)e_{12345} \,, \tag{5.1}$$

and a basis in the space of Killing spinors such that (3.10) is satisfied is

$$\eta^{\bar{\alpha}\bar{\beta}} = e_{\alpha\beta} \,, \quad \eta^{\bar{\alpha}} = e_{\alpha 5} \,,$$
$$\eta^{\alpha} = \frac{1}{6}\epsilon^{\alpha\beta_1\beta_2\beta_3}e_{\beta_1\beta_2\beta_3 5} \,, \quad \eta^+ = (n - \ell + im)1 - (n + \ell + im)e_{1234} \,. \tag{5.2}$$

T^2 is constrained by

$$(T^2)_{+-} = (T^2)_{-\mu} = (T^2)_{-\bar{\mu}} = 0 \,,$$
$$(T^2)_{+\mu} = -\frac{1}{8}(n - \ell + im)u_\mu \,, \qquad\qquad (T^2)_{+\bar{\mu}} = -\frac{1}{8}(n + \ell + im)u_{\bar{\mu}} \,,$$
$$(T^2)_{\mu\nu} = -\frac{1}{16}(n - \ell + im)\epsilon_{\mu\nu}{}^{\bar{\beta}_1\bar{\beta}_2}u_{\bar{\beta}_1\bar{\beta}_2} \,, \quad (T^2)_{\mu\bar{\nu}} = \frac{1}{8}\big((n + im)^2 - \ell^2\big)u_+\delta_{\mu\bar{\nu}} \,,$$
$$(T^2)_{\bar{\mu}\bar{\nu}} = \frac{1}{8}(n + \ell + im)u_{\bar{\mu}\bar{\nu}} \,. \tag{5.3}$$

The analysis proceeds depending on whether or not $(n + im)^2 - \ell^2$ vanishes. There are three cases but two of them are related by a $\mathrm{Spin}(9,1)$ transformation. So there are two independent cases to consider.

5.1 Generic solutions $((n + im)^2 - \ell^2 \neq 0)$

In this case there are no restrictions on the spacetime functions n, m and ℓ. It is then straightforward to see, using the same reasoning as in the $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$ analysis, that all components of u_r vanish except for $u_{+A,B}$, where $A = (\alpha, \bar{\alpha})$, $B = (\beta, \bar{\beta})$, and

$$(T^2{}_{+\alpha})_{+\beta} = -\frac{1}{8}(n - \ell + im)u_{+\alpha,\beta} \,, \quad (T^2{}_{+\alpha})_{+\bar{\beta}} = -\frac{1}{8}(n + \ell + im)u_{+\alpha,\bar{\beta}} \,,$$
$$(T^2{}_{+\bar{\alpha}})_{+\beta} = -\frac{1}{8}(n - \ell + im)u_{+\bar{\alpha},\beta} \,, \quad (T^2{}_{+\bar{\alpha}})_{+\bar{\beta}} = -\frac{1}{8}(n + \ell + im)u_{+\bar{\alpha},\bar{\beta}} \,. \tag{5.4}$$

Similarly, it turns out that all components of T^4 are forced to vanish by the above constraints with the exception of

$$(T^4)_{+\mu\nu\rho} = -\frac{1}{4}(n - \ell + im)u_{\bar{\alpha}}\epsilon^{\bar{\alpha}}{}_{\mu\nu\rho} \,,$$
$$(T^4)_{+\mu\nu\bar{\rho}} = \frac{1}{8}(n - \ell + im)\big(u_\mu\delta_{\nu\bar{\rho}} - u_\nu\delta_{\mu\bar{\rho}}\big) \,,$$
$$(T^4)_{+\mu\bar{\nu}\bar{\rho}} = \frac{1}{8}(n + \ell + im)\big(\delta_{\mu\bar{\rho}}u_{\bar{\nu}} - \delta_{\mu\bar{\nu}}u_{\bar{\rho}}\big) \,,$$
$$(T^4)_{+\bar{\mu}\bar{\nu}\bar{\rho}} = -\frac{1}{4}(n + \ell + im)u_\alpha\epsilon^{\alpha}{}_{\bar{\mu}\bar{\nu}\bar{\rho}} \,. \tag{5.5}$$

As $(T^4{}_{+A_1})_{+A_2A_3A_4}$ is totally antisymmetric in A_i, self-duality implies that $(T^4{}_{+\alpha})_{+\beta\rho\sigma} = 0$, and hence $u_{+\alpha,\bar{\beta}} = 0$. Therefore $(T^2{}_{+\alpha})_{+\bar{\beta}} = 0$, and hence $(T^2{}_{+\bar{\alpha}})_{+\beta} = 0$ also implies $u_{+\bar{\alpha},\beta} = 0$.

Furthermore, we also have

$$(T^4{}_{+\mu})_{+\alpha\beta}{}^{\beta} = \frac{3}{8}(n - \ell + im)u_{+\mu,\alpha} \,. \tag{5.6}$$

As the left-hand side of this expression must vanish by self-duality, we find $u_{+\alpha,\beta} = 0$. Hence $(T^2{}_{+\alpha})_{+\beta} = 0$, and so $(T^2{}_{+\bar\alpha})_{+\bar\beta} = 0$ also implies that $u_{+\bar\alpha,\bar\beta} = 0$. Therefore all components of the u_r vanish, so all components of T^2 and T^4 are constrained to vanish as well.

5.2 Pure spinor solution ($(n + im)^2 - \ell^2 = 0$)

There are two pure spinor cases that one can consider depending on whether $m = 0$, $n = \ell \neq 0$ or $m = 0$, $n = -\ell \neq 0$. The normal directions are either $\nu = e_{1234}$ or $\nu = 1$, respectively. However, these two normals are related by a Spin$(9,1)$ transformation. So it suffices to consider one of the two cases as the other will follow by virtue of the Spin$(9,1)$ gauge symmetry of the Killing spinor equations. So let us investigate the case $m = 0$, $n = \ell$. Then (5.3) implies that $(T^2)_{+\alpha} = 0$. Therefore, $(T^2)_{+\bar\alpha} = 0$, so $u_{\bar\alpha} = 0$. Furthermore, $(T^2)_{\alpha\beta} = 0$, so $(T^2)_{\bar\alpha\bar\beta} = 0$ also, and therefore $u_{\bar\alpha\bar\beta} = 0$. These constraints are sufficient to fix $T^2 = 0$, however u_+ and u_α are not fixed by constraints involving T^2.

It is straightforward to see that the only non-vanishing components of T^4 are given by

$$(T^4)_{+\bar\alpha\bar\beta\bar\lambda} = \frac{n}{2}\epsilon_{\bar\alpha\bar\beta\bar\lambda}{}^\rho u_\rho\,, \quad (T^4)_{\bar\alpha\bar\beta\bar\lambda\bar\sigma} = -n^2 u_+ \epsilon_{\bar\alpha\bar\beta\bar\lambda\bar\sigma} \,. \tag{5.7}$$

To proceed, note that the self-duality constraint fixes $(T^4{}_{+\bar\sigma})_{+\bar\alpha\bar\beta\bar\lambda} = 0$, so $u_{+\bar\beta,\alpha} = 0$. Also, $(T^4{}_{+\sigma})_{+\bar\alpha\bar\beta\bar\lambda} = -(T^4{}_{+\bar\alpha})_{+\sigma\bar\beta\bar\lambda} = 0$, so $u_{+\beta,\alpha} = 0$. Furthermore $(T^4{}_{[\mu\nu})_{\bar\alpha\bar\beta\bar\lambda\bar\sigma]} = 0$ which implies $(T^4{}_{\mu\nu})_{\bar\alpha\bar\beta\bar\lambda\bar\sigma} = 0$ and hence $u_{\mu\nu,+} = 0$. Also, $(T^4{}_{[-\nu})_{\bar\alpha\bar\beta\bar\lambda\bar\sigma]} = 0$ implies $(T^4{}_{-\nu})_{\bar\alpha\bar\beta\bar\lambda\bar\sigma} = 0$, so $u_{-\alpha,+} = 0$.

Next, consider the following relation implied by self-duality:

$$(T^4{}_{+[\nu})_{\bar\alpha\bar\beta\bar\lambda\bar\sigma]} = -\frac{1}{6}\epsilon_{\bar\alpha\bar\beta\bar\lambda\bar\sigma}\epsilon_\nu{}^{\bar\lambda_1\bar\lambda_2\bar\lambda_3}(T^4{}_{+[-)+\bar\lambda_1\bar\lambda_2\bar\lambda_3]} \,. \tag{5.8}$$

This implies that

$$nu_{+\alpha,+} = -\frac{1}{2}u_{+-,\alpha} \,. \tag{5.9}$$

However, $(T^4{}_{+-})_{+\bar\lambda_1\bar\lambda_2\bar\lambda_3} = -(T^4{}_{+\bar\lambda_1})_{+-\bar\lambda_2\bar\lambda_3} = 0$, which implies that $u_{+-,\alpha} = 0$, so $u_{+\alpha,+} = 0$ as well. Also, $(T^4{}_{[-\rho})_{+\bar\alpha\bar\beta\bar\lambda]} = 0$, which implies $(T^4{}_{-\rho})_{+\bar\alpha\bar\beta\bar\lambda} = 0$ and so $u_{-\alpha,\beta} = 0$.

Also note that $(T^4{}_{-(\bar\alpha})_{\bar\beta)\bar\rho\bar\sigma\bar\lambda} = -(T^4{}_{\bar\rho(\bar\alpha})_{\bar\beta)-\bar\sigma\bar\lambda} = 0$, so

$$u_{-\bar\alpha,+}\epsilon_{\bar\beta\bar\rho\bar\sigma\bar\lambda} + u_{-\bar\beta,+}\epsilon_{\bar\alpha\bar\rho\bar\sigma\bar\lambda} = 0 \,. \tag{5.10}$$

Contracting this expression with $\epsilon^{\bar\beta\bar\rho\bar\sigma\bar\lambda}$ yields $u_{-\bar\alpha,+} = 0$.

Next consider $(T^4{}_{-(+})_{\bar\alpha)\bar\beta\bar\lambda\bar\sigma} = -(T^4{}_{\bar\beta(+})_{\bar\alpha)-\bar\lambda\bar\sigma} = 0$. This implies that

$$n^2 u_{-+,+}\epsilon_{\bar\alpha\bar\beta\bar\lambda\bar\sigma} - \frac{n}{2}u_{-\bar\alpha,\rho}\epsilon_{\bar\beta\bar\lambda\bar\sigma}{}^\rho = 0 \tag{5.11}$$

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and on contracting with $\epsilon^{\bar{\beta}\bar{\lambda}\bar{\sigma}}{}_\mu$, we find

$$u_{-\bar{\alpha},\mu} = -2n\delta_{\bar{\alpha}\mu}u_{-+,+} \ . \tag{5.12}$$

However, self-duality implies that $(T^4{}_{-[+})_{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\sigma}]} = 0$, which when combined with (5.12) is sufficient to constrain $u_{-+,+} = 0$ and hence $u_{-\bar{\alpha},\mu} = 0$ as well.

Next, note that $(T^4{}_{\mu(\bar{\nu})\bar{\alpha})\bar{\beta}\bar{\lambda}\bar{\rho}} = -(T^4{}_{\bar{\beta}(\bar{\nu})\bar{\alpha})\mu\bar{\lambda}\bar{\rho}} = 0$, hence

$$u_{\mu\bar{\nu},+}\epsilon_{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\rho}} + u_{\mu\bar{\alpha},+}\epsilon_{\bar{\nu}\bar{\beta}\bar{\lambda}\bar{\rho}} = 0 \ . \tag{5.13}$$

On contracting this identity with $\epsilon^{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\rho}}$ we find $u_{\mu\bar{\nu},+} = 0$.

The constraint $(T^4{}_{+(\bar{\mu})\bar{\alpha})\bar{\beta}\bar{\lambda}\bar{\sigma}} = -(T^4{}_{\bar{\beta}(\bar{\mu})\bar{\alpha})+\bar{\lambda}\bar{\sigma}}$ implies, on contracting with $\epsilon^{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\sigma}}$, that

$$6nu_{+\bar{\mu},+} = -\delta^{\rho\bar{\beta}}u_{\bar{\beta}\bar{\mu},\rho} \tag{5.14}$$

and furthermore the self-duality constraint $(T^4{}_{\bar{\mu}[+})_{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\sigma}]} = 0$ implies, on contracting with $\epsilon^{\bar{\alpha}\bar{\beta}\bar{\lambda}\bar{\sigma}}$, that

$$24n^2 u_{+\bar{\mu},+} - 12n\delta^{\rho\bar{\beta}}u_{\bar{\beta}\bar{\mu},\rho} = 0 \ . \tag{5.15}$$

This constraint, together with (5.14) implies that $u_{+,\bar{\mu},+} = 0$ and $\delta^{\rho\bar{\beta}}u_{\bar{\beta}\bar{\mu},\rho} = 0$. Next note that $(T^4{}_{\bar{\mu}(\bar{\nu})\bar{\alpha})\bar{\beta}\bar{\rho}\bar{\sigma}} = -(T^4{}_{\bar{\beta}(\bar{\nu})\bar{\alpha})\bar{\mu}\bar{\rho}\bar{\sigma}}$. Contracting this constraint with $\epsilon^{\bar{\alpha}\bar{\beta}\bar{\rho}\bar{\sigma}}$ gives $u_{\bar{\mu}\bar{\nu},+} = 0$.

Combining all of these constraints fixes all components of u_+ to vanish. To fix the remaining components of u_α, note that $(T^4{}_{\bar{\mu}(\bar{\nu})+)\bar{\alpha}\bar{\beta}\bar{\lambda}} = -(T^4{}_{\bar{\alpha}(\bar{\nu})+)\bar{\mu}\bar{\beta}\bar{\lambda}}$ implies that

$$\epsilon_{\bar{\alpha}\bar{\beta}\bar{\lambda}}{}^\rho u_{\bar{\mu}\bar{\nu},\rho} = -\epsilon_{\bar{\mu}\bar{\beta}\bar{\lambda}}{}^\rho u_{\bar{\alpha}\bar{\nu},\rho} \tag{5.16}$$

and on contracting this expression with $\epsilon^{\bar{\alpha}\bar{\beta}\bar{\lambda}}{}_\sigma$ and using the constraint $\delta^{\rho\bar{\beta}}u_{\bar{\beta}\bar{\mu},\rho} = 0$ which we have already obtained, we find $u_{\bar{\mu}\bar{\nu},\sigma} = 0$.

Next, note that the constraint $(T^4{}_{\mu(\bar{\nu})+)\bar{\alpha}\bar{\beta}\bar{\lambda}} = -(T^4{}_{\bar{\alpha}(\bar{\nu})+)\mu\bar{\beta}\bar{\lambda}} = 0$ together with $u_+ = 0$ implies that $(T^4{}_{\mu\bar{\nu}})_{+\bar{\alpha}\bar{\beta}\bar{\lambda}} = 0$, so $u_{\mu\bar{\nu},\rho} = 0$. Finally, $(T^4{}_{\mu(\nu)+)\bar{\alpha}\bar{\beta}\bar{\lambda}} = -(T^4{}_{\bar{\alpha}(\nu)+)\mu\bar{\beta}\bar{\lambda}} = 0$ together with $u_+ = 0$ imply that $(T^4{}_{\mu\nu})_{+\bar{\alpha}\bar{\beta}\bar{\lambda}} = 0$, so $u_{\mu\nu,\rho} = 0$.

These constraints are then sufficient to fix $u_\alpha = 0$, and hence all components of u_r vanish, as do T^2 and T^4.

6. G_2-invariant normal

The normal spinor can be chosen as

$$\nu = n(e_5 + e_{12345}) + im(e_1 + e_{234}) \ . \tag{6.1}$$

By using a gauge transformation of the form $e^{f\Gamma_{+-}}$ for real f, we can without loss of generality set $m = \pm n$, and so we take the normal spinor direction as

$$\nu = e_5 + e_{12345} \pm i(e_1 + e_{234}) \ . \tag{6.2}$$

A basis of spinors compatible with (3.10) is

$$\eta^- = e_{15} + e_{2345} \mp i(1 + e_{1234})\,, \qquad \eta^+ = 1 - e_{1234}\,,$$

$$\eta^1 = e_{15} - e_{2345}\,, \qquad\qquad \eta^{1\bar{p}} = e_{1p}\,, \qquad\qquad \eta^{1p} = \frac{1}{2}\epsilon_{pqr}e_{qr}\,,$$

$$\eta^{\bar{p}} = e_{p5}\,, \qquad\qquad\qquad \eta^p = \frac{1}{2}\epsilon_{pqr}e_{qr} \wedge e_{15}\,, \qquad\qquad (6.3)$$

where $p,q,r = 1,2,3$. We then find the following constraints on T^2:

$$(T^2)_{+-} = \pm\frac{i}{4}u_-\,, \qquad\qquad (T^2)_{+1} = -\frac{1}{8}(u_- - u_1)\,, \qquad (T^2)_{+\bar{1}} = -\frac{1}{8}(u_- + u_1)\,,$$

$$(T^2)_{+p} = \frac{1}{8}u_p\,, \qquad\qquad\qquad (T^2)_{+\bar{p}} = -\frac{1}{8}u_{\bar{p}}\,,$$

$$(T^2)_{-1} = -\frac{1}{8}(-u_- \mp iu_+)\,, \qquad (T^2)_{-\bar{1}} = -\frac{1}{8}(-u_- \pm iu_+)\,, \qquad (T^2)_{-p} = \pm\frac{i}{8}u_{1p}\,,$$

$$(T^2)_{-\bar{p}} = \mp\frac{i}{8}u_{1\bar{p}}\,,$$

$$(T^2)_{1\bar{1}} = -\frac{1}{8}(\pm iu_1 - u_+)\,, \qquad (T^2)_{1p} = -\frac{1}{8}u_{1p}\,, \qquad\qquad (T^2)_{1\bar{p}} = \mp\frac{i}{8}u_{\bar{p}}\,,$$

$$(T^2)_{\bar{1}p} = \pm\frac{i}{8}u_p\,, \qquad\qquad (T^2)_{\bar{1}\bar{p}} = \frac{1}{8}u_{1\bar{p}}\,,$$

$$(T^2)_{pq} = -\frac{1}{8}\epsilon_{pq}{}^{\bar{r}}(u_{1\bar{r}} \pm iu_{\bar{r}})\,, \qquad (T^2)_{p\bar{q}} = -\frac{1}{8}\delta_{p\bar{q}}(-u_+ \mp iu_1)\,,$$

$$(T^2)_{\bar{p}\bar{q}} = -\frac{1}{8}\epsilon_{\bar{p}\bar{q}}{}^{r}(-u_{1r} \mp iu_r)\,. \qquad\qquad (6.4)$$

These constraints imply that

$$u_- = \mp 4i(T^2)_{+-}\,, \quad u_1 = -4((T^2)_{+\bar{1}} - (T^2)_{+1})\,, \qquad u_p = 8(T^2)_{+p}\,,$$

$$u_{\bar{p}} = -8(T^2)_{+\bar{p}}\,, \quad u_+ = \pm 4i((T^2)_{-\bar{1}} - (T^2)_{-1})\,, \quad u_{1p} = -8(T^2)_{1p}\,,$$

$$u_{1\bar{p}} = 8(T^2)_{\bar{1}\bar{p}}\,. \qquad\qquad (6.5)$$

Substituting (6.5) back into (6.4) gives the constraints

$$(T^2)_{+1} + (T^2)_{+\bar{1}} = \pm i(T^2)_{+-}\,, \qquad\qquad\qquad (T^2)_{-1} + (T^2)_{-\bar{1}} = \mp i(T^2)_{+-}\,,$$

$$(T^2)_{-p} = \mp i(T^2)_{1p}\,, \qquad\qquad\qquad\qquad (T^2)_{-\bar{p}} = \mp i(T^2)_{\bar{1}\bar{p}}\,,$$

$$(T^2)_{1\bar{1}} = \pm\frac{i}{2}\big((T^2)_{+\bar{1}} - (T^2)_{+1} + (T^2)_{-\bar{1}} - (T^2)_{-1}\big)\,,$$

$$(T^2)_{1\bar{p}} = \pm i(T^2)_{+\bar{p}}\,, \qquad\qquad\qquad\qquad (T^2)_{\bar{1}p} = \pm i(T^2)_{+p}\,,$$

$$(T^2)_{pq} = \epsilon_{pq}{}^{\bar{r}}(-(T^2)_{\bar{1}\bar{r}} \pm i(T^2)_{+\bar{r}})\,,$$

$$(T^2)_{p\bar{q}} = \pm\frac{i}{2}\delta_{p\bar{q}}\big((T^2)_{-\bar{1}} - (T^2)_{-1} - (T^2)_{+\bar{1}} + (T^2)_{+1}\big)\,,$$

$$(T^2)_{\bar{p}\bar{q}} = \epsilon_{\bar{p}\bar{q}}{}^{r}(-(T^2)_{1r} \pm i(T^2)_{+r})\,. \qquad\qquad (6.6)$$

These constraints can be rewritten in terms of irreducible G_2 representations[1] as

$$(T^2)_{+\underline{1}} = \pm\frac{i}{\sqrt{2}}(T^2)_{+-}, \quad (T^2)_{-\underline{1}} = \mp\frac{i}{\sqrt{2}}(T^2)_{+-},$$

$$(T^2)_{\underline{1}i} = \pm\frac{i}{\sqrt{2}}((T^2)_{+i} + (T^2)_{-i}),$$

$$(\Pi_{\mathbf{7}}T^2)_i := \varphi_i{}^{jk}(T^2)_{jk} = \pm 3\sqrt{2}i((T^2)_{+i} - (T^2)_{-i}),$$

$$(\Pi_{\mathbf{14}}T^2)_{ij} := \frac{2}{3}\left(\frac{1}{4}\star\varphi_{ij}{}^{kl}(T^2)_{kl} + (T^2)_{ij}\right) = 0, \tag{6.7}$$

where the underlined 1 denotes a real index. By taking the complex conjugate of these expressions, and using the fact that T^2_{MN} is real, one immediately finds that all components of T^2_{MN} are put to zero. This implies, through (6.5), that all components of u_r vanish.

Note that throughout this reasoning, in contrast to the analysis of the $\mathrm{Spin}(7) \ltimes \mathbb{R}^8$ and $\mathrm{SU}(4) \ltimes \mathbb{R}^8$ cases, we have not made use of the algebraic constraints on T^2 given in (3.3) and (3.4); only the fact that T^2 is real has been used.

To summarize, we have shown that all components of the u_r vanish, so all components of T^2 and T^4 also vanish. This yields $\mathcal{R} = 0$ in this case as well. We therefore conclude that for the $N > 28$ IIB backgrounds $\mathcal{R} = 0$ and they are thus locally isometric to maximally supersymmetric backgrounds.

7. Discrete quotients

We have shown that all $N > 28$ supersymmetric IIB backgrounds are locally maximally supersymmetric. So it remains to exclude the possibility that $28 < N < 32$ backgrounds can be constructed by discrete quotients of maximally supersymmetric ones. The maximally supersymmetric backgrounds of IIB supergravity have been classified [12]. It has been found that they are locally isometric to Minkowski space $\mathbb{R}^{9,1}$, $AdS_5 \times S^5$ [13] and the maximally supersymmetric plane wave [14]. Considering the simply connected maximally supersymmetric backgrounds, which we collectively denote as $\tilde{M}$, one chooses a discrete subgroup D of their symmetry group S, and constructs new solutions by taking the quotient of $\tilde{M}$ with D, $\tilde{M}/D$. Such backgrounds are solutions of the field equations and depending on the choice of D typically preserve less supersymmetry than $\tilde{M}$. So the task is to find whether there are subgroups D such that $\tilde{M}/D$ preserves $28 < N < 32$ supersymmetries. The linearity of the Killing spinor equations of IIB supergravity for backgrounds with $P = G = 0$ over the complex numbers excludes the possibility of $\tilde{M}/D$ preserving an odd number of supersymmetries. So to prove that there are no new supersymmetric backgrounds with $N > 28$, we have to show that there are no $N = 30$ quotients of maximally supersymmetric backgrounds.

The task of proving that there are no subgroups $D \subset S$ of the symmetry group of simply connected maximally supersymmetric IIB backgrounds $\tilde{M}$ for which $\tilde{M}/D$ preserves 30 supersymmetries is simplified in two ways. First it has been shown in [4] that, without

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[1]This can be seen as a consistency check of the calculation.

loss of generality, one can consider only cyclic subgroups D as the remaining possibilities can be reduced to this case. In addition, it suffices to take the generator α of the cyclic group, $D = <\alpha>$, to lie in the image of the exponential map of S. Therefore $\alpha = e^X$, where X is an element of the Lie algebra of S. Since D is specified up to a conjugation in S, it suffices to consider the normal forms of X up to the action of the adjoint map of S. This is a straightforward task for compact groups but for non-compact ones, like S, there are several possibilities as has been emphasized in [7].

One continues the computation by considering the lift $\hat{\alpha}$ of the generator α to the spin bundle and by computing the number of invariant Killing spinors under the action of $\hat{\alpha}$. The number of invariant Killing spinors is the number of supersymmetries preserved by $\tilde{M}/D$.

One difference that arises in the IIB case, in comparison with the cases investigated in [7, 8], is that the group action should be lifted to a $Spin_c(9,1) = \mathrm{Spin}(9,1) \times_{\mathbb{Z}_2} \mathrm{U}(1)$ rather than a $\mathrm{Spin}(9,1)$ bundle. This is equivalent to allowing an additional phase in the lift $\hat{\alpha}$ of the generator α of D along the $\mathrm{U}(1)$ direction. This additional phase is similar to that which appears in the context of supersymmetric backgrounds in three-dimensional supergravities as the holonomy of a flat $\mathrm{U}(1)$ connection [21]. It is known that the inclusion of the $\mathrm{U}(1)$ phase changes the number of supersymmetries preserved by a background. Such backgrounds are the stringy cosmic strings [22], the D7-branes [23] and the conical purely gravitational domain walls of [24].

7.1 Discrete quotients of $\mathbb{R}^{9,1}$

Let us begin with the flat space case. The translations do not reduce supersymmetry so they are not appropriate for the construction of $N < 32$ backgrounds. On the other hand discrete quotients with elements of the isometry group $\mathrm{SO}(9,1)$ of $\mathbb{R}^{9,1}$ do not preserve all supersymmetry. So consider the generator $\alpha = \exp X$, $X \in \mathfrak{so}(9,1)$, of the cyclic group. Then up to a conjugation, one has that either

$$X = -\theta_0 e^0 \wedge e^5 + \theta_1 e^1 \wedge e^6 + \theta_2 e^2 \wedge e^7 + \theta_3 e^3 \wedge e^8 + \theta_4 e^4 \wedge e^9 \,, \tag{7.1}$$

or

$$X = -(e^0 - e^5) \wedge e^9 + \theta_1 e^1 \wedge e^6 + \theta_2 e^2 \wedge e^7 + \theta_3 e^3 \wedge e^8 \,. \tag{7.2}$$

In the former case, α lifts to the element

$$\hat{\alpha} = \exp\left(\frac{1}{2}(\theta_0\Gamma_{05} + \theta_1\Gamma_{16} + \theta_2\Gamma_{27} + \theta_3\Gamma_{38} + \theta_4\Gamma_{49}) + i\psi\right) \tag{7.3}$$

of $Spin_c(9,1)$, where ψ is the angle along the $\mathrm{U}(1)$ direction. Since $\Gamma_{05}, \Gamma_{16}, \Gamma_{27}, \Gamma_{38}$ and Γ_{49} are commuting with $-(\Gamma_{05})^2 = (\Gamma_{16})^2 = (\Gamma_{27})^2 = (\Gamma_{38})^2 = (\Gamma_{49})^2 = -1_{16\times16}$, the Weyl representation decomposes in subspaces which are the eigenspaces of the above matrices, i.e.

$$\Delta_{\mathbf{16}} = \oplus_{\sigma_0,\ldots,\sigma_4} W_{\sigma_0\ldots\sigma_4} \,, \tag{7.4}$$

where $\sigma_0, \ldots, \sigma_4$ are signs restricted by the chirality condition to satisfy $\sigma_0\sigma_1\sigma_2\sigma_3\sigma_4 = 1$. Therefore acting on the subspace $W_{\sigma_0\ldots\sigma_4}$, one has

$$\hat{\alpha}(\sigma_0, \ldots, \sigma_4) = \exp\left(\frac{1}{2}(\sigma_0\theta_0 + i\sigma_1\theta_1 + i\sigma_2\theta_2 + i\sigma_3\theta_3 + i\sigma_4\theta_4) + i\psi\right) . \qquad (7.5)$$

Now to find the supersymmetry preserved by a discrete quotient constructed from α, one has to determined the spinors which are left invariant under the action of $\hat{\alpha}$. This in particular implies that there must be angles or boosts such that

$$\exp\left(\frac{1}{2}(\sigma_0\theta_0 + i\sigma_1\theta_1 + i\sigma_2\theta_2 + i\sigma_3\theta_3 + i\sigma_4\theta_4) + i\psi\right) = 1 , \qquad (7.6)$$

for some choice of signs σ. Taking the complex conjugate, we conclude that

$$\theta_0 = 0 . \qquad (7.7)$$

Moreover, since we require at least 30 supersymmetries to be preserved, there are $\sigma_0, \sigma_1, \ldots, \sigma_4$ such that if $\hat{\alpha}(\sigma_0, \sigma_1, \ldots, \sigma_4) = 1$, then $\hat{\alpha}(\sigma_0, \bar{\sigma}_1, \ldots, \bar{\sigma}_4) = 1$ for $\bar{\sigma} = -\sigma$. Observe that this is consistent with the chirality restriction. Using this and $\theta_0 = 0$, we find that

$$\hat{\alpha}(\sigma_0, \sigma_1, \ldots, \sigma_4)\hat{\alpha}(\sigma_0, \bar{\sigma}_1, \ldots, \bar{\sigma}_4) = e^{2i\psi} = 1 \qquad (7.8)$$

and so $\psi = n\pi$, $n \in \mathbb{Z}$. To preserve 30 real supersymmetries, we have to impose 15 conditions over the complex numbers. But since $e^{i\psi} = \pm 1$, if $\hat{\alpha}(\sigma_0, \sigma_1, \ldots, \sigma_4) = 1$, then $(\hat{\alpha}(\sigma_0, \sigma_1, \ldots, \sigma_4))^* = \hat{\alpha}(\sigma_0, \bar{\sigma}_1, \ldots, \bar{\sigma}_4) = 1$. Therefore one can impose an even number of conditions each time. As a consequence supersymmetry can reduce only mod 2 over the complex numbers or mod 4 over the reals. This in particular excludes the existence of discrete quotients with $N = 30$ supersymmetries.

It remains to see whether the lift of (7.2) can preserve 30 supersymmetries. In this case, we have

$$\hat{\alpha} = \exp\left(\frac{1}{2}[(\Gamma_0 + \Gamma_5)\Gamma_9 + \theta_1\Gamma_{16} + \theta_2\Gamma_{27} + \theta_3\Gamma_{38}] + i\psi\right) . \qquad (7.9)$$

Observe that this can be rewritten as

$$\hat{\alpha} = \rho\left[1 + \frac{1}{2}(\Gamma_0 + \Gamma_5)\Gamma_9\right] , \quad \rho = \exp\left(\frac{1}{2}[\theta_1\Gamma_{16} + \theta_2\Gamma_{27} + \theta_3\Gamma_{38}] + i\psi\right) . \qquad (7.10)$$

Now the invariance condition can be written as

$$\rho\,\epsilon_- = \epsilon_- , \quad \rho\,\epsilon_+ + \rho\,\Gamma_{09}\,\epsilon_- = \epsilon_+ , \qquad (7.11)$$

where we have decomposed the spinors in the eigenspaces $V_- \oplus V_+$ of Γ_{05} as $\Gamma_{05}\epsilon_\pm = \pm\epsilon_\pm$. To preserve 30 supersymmetries at least 7 complex spinors in V_- must satisfy the first equation for ϵ_-. Since Γ_{09} is invertible this would imply that the second invariance equation cannot be satisfied on an at least seven-dimensional subspace of V_+. So there is no invariant complex 15-dimensional subspace in $V_- \oplus V_+$ which is required to preserve 30 supersymmetries. Combining this with the result in the previous case, one concludes that there are no quotients of flat space that can preserve 30 supersymmetries.

7.2 Discrete quotients of $AdS_5 \times S^5$

The isometry group of this background is $SO(4,2) \times SO(6)$. Therefore one can choose $\alpha = e^{X+Y}$ where $X \in \mathfrak{so}(4,2)$ and $Y \in \mathfrak{so}(6)$. In addition, it can be arranged such that $\mathrm{Spin}(4,2) \times \mathrm{Spin}(6)$ acts on the Weyl representation of $\mathrm{Spin}(9,1)$ as $\Delta^-_{\mathrm{Spin}(4,2)} \otimes \Delta^-_{\mathrm{Spin}(6)}$, where $\Delta^-_{\mathrm{Spin}(4,2)}$ $\Delta^-_{\mathrm{Spin}(6)}$ are the anti-chiral Weyl representations of $\mathrm{Spin}(4,2)$ and $\mathrm{Spin}(6)$, respectively. Therefore the lifted element $\hat{\alpha}$ of α can be written as

$$\hat{\alpha} = e^{X+Y+i\psi}, \tag{7.12}$$

where X and Y are Clifford algebra elements and ψ is an additional angle because of the $Spin_c(9,1)$ nature of the IIB spinors.

There is a unique normal form for Y up to a $\mathrm{Spin}(6)$ conjugation which we can take to be

$$Y = \frac{1}{2}(\theta_1 \gamma_{12} + \theta_2 \gamma_{34} + \theta_3 \gamma_{56}), \tag{7.13}$$

where θ_1, θ_2 and θ_3 are $SO(6)$ rotation angles, and γ_i are $\mathrm{Spin}(6)$ gamma matrices. Moreover $\Delta^-_{\mathrm{Spin}(6)}$ can be decomposed in four complex one-dimensional spaces in which case one has that

$$Y = \frac{i}{2}(\sigma_1 \theta_1 + \sigma_2 \theta_2 + \sigma_3 \theta_3), \tag{7.14}$$

where $\sigma_1 \sigma_2 \sigma_3 = 1$, $\sigma_i = \pm 1$, due to the chirality restriction.

There are 25 possible normal forms for X up to $SO(4,2)$ conjugations. These have be tabulated in [7] and we shall not repeat them here. As a consequence, we have to investigate 25 cases to see whether there are quotients of $AdS_5 \times S^5$ that preserve 30 supersymmetries. In what follows, we shall use the numbering of cases as in [7] but we have made some adjustments in the notation because of our different spinor conventions.

7.2.1 Cases 1, 2, 4, 10, 11, 12, 16, 24 and 25

In case 24, the normal form for X can be taken as

$$X = \frac{1}{2}(\zeta_1 \tilde{\gamma}_{05} + \zeta_2 \tilde{\gamma}_{12} + \zeta_3 \tilde{\gamma}_{34}), \tag{7.15}$$

where 0 and 5 are the time-like directions and the rest are spacelike, $\tilde{\gamma}$ are the gamma matrices of $\mathrm{Spin}(4,2)$ and ζ_i are angles. Decomposing $\Delta^-_{\mathrm{Spin}(4,2)}$ in one-dimensional complex representations we get that

$$X = \frac{i}{2}(s_1 \zeta_1 + s_2 \zeta_2 + s_3 \zeta_3), \tag{7.16}$$

where $s_1 s_2 s_3 = 1$ because of the chirality condition and $s_a = \pm 1$. Therefore the lifted element $\hat{\alpha}$ of α is

$$\hat{\alpha}(s_1, s_2, \sigma_1, \sigma_2) = e^{\frac{i}{2}(\sum_a s_a \zeta_a + \sum_i \sigma_i \theta_i) + i\psi}. \tag{7.17}$$

To preserve 30 supersymmetries $\hat{\alpha}(s, \sigma) = 1$ for 15 out of 16 choices of signs for s_a and σ_i subject to the chirality conditions $s_1 s_2 s_3 = 1$ and $\sigma_1 \sigma_2 \sigma_3 = 1$. Without loss of generality let us assume that $\hat{\alpha}(s, \sigma) = 1$ unless when $\sigma_1 = \sigma_2 = s_1 = s_2 = -1$ for which we take $\hat{\alpha}(-1, -1, -1, -1) \neq 1$. Since $\hat{\alpha}(-1, -1, 1, 1) = \hat{\alpha}(1, 1, 1, 1) = 1$, then

$$(\hat{\alpha}(-1, -1, 1, 1))^* \hat{\alpha}(1, 1, 1, 1) = e^{-i\zeta_1 - i\zeta_2} = 1 \ . \tag{7.18}$$

Then observe that

$$e^{-i\zeta_1 - i\zeta_2} \hat{\alpha}(1, 1, -1, -1) = \hat{\alpha}(-1, -1, -1, -1) = 1 , \tag{7.19}$$

which is a contradiction. Therefore if one assumes that $\hat{\alpha}$ preserves 30 supersymmetries, then one can show that it preserves 32. So there are no such $N = 30$ supersymmetric quotients of $AdS_5 \times S^5$.

Before we proceed to other cases, notice that the same conclusion holds if one of the angles ζ and/or one of the angles θ vanish. This can be shown in exactly the same way as the general case above. In addition, if either two or more angles ζ vanish or two or more angles θ vanish, then the decomposition of the Weyl representation of $\mathrm{Spin}(9,1)$ with respect to $X + Y$ will be in subspaces of complex dimension more than one. Consequently, the invariant subspaces will have dimension either 32 and all supersymmetry will be preserved or always less than 30. Therefore one concludes that there are no $N = 30$ quotients even if one or more angles ζ, θ vanish.

In the case 25 of [7], the normal form of X give rise to

$$X = \zeta_1 \tilde{\gamma}_{01} + \zeta_2 \tilde{\gamma}_{52} + \zeta_3 \tilde{\gamma}_{34} , \tag{7.20}$$

which after decomposing the Weyl representation in one-dimensional complex subspaces one gets

$$\hat{\alpha}(s_1, s_2, \sigma_1, \sigma_2) = e^{\frac{1}{2}(s_1 \zeta_1 + s_2 \zeta_2 + i s_3 \zeta_3 + i \sum_i \sigma_i \theta_i) + i\psi} , \tag{7.21}$$

where the signs s and σ obey the chirality conditions as in the previous case. In this case ζ_1 and ζ_2 are boosts. If for some signs $\hat{\alpha}(s_1, s_2, \sigma_1, \sigma_2) = 1$, then $(\hat{\alpha}(s_1, s_2, \sigma_1, \sigma_2))^* = 1$, which implies that

$$e^{s_1 \zeta_1 + s_2 \zeta_2} = 1 \ . \tag{7.22}$$

There are four possible uncorrelated choices for the signs s_1 and s_2. To preserve $N = 30$ supersymmetry for three of these choices the above condition must hold. Without loss of generality one can take

$$e^{\zeta_1 + \zeta_2} = e^{\zeta_1 - \zeta_2} = 1 \ . \tag{7.23}$$

This in turn gives $\zeta_1 = \zeta_2 = 0$. Consequently this reduces to (7.17) with two vanishing angles. As we have shown such quotients do not preserve 30 supersymmetries. The same conclusion holds if one or more of the boosts or rotation angles vanishes. Consequently, one can also conclude that the normal forms of the cases 1,2,4,10,11,12 and 16 [7] do not give quotients which preserve 30 supersymmetries.

7.2.2 Cases 3, 5, 14, 15 and 17

In case 14, the lifted element is

$$\hat{\alpha} = \rho \, e^{\frac{1}{2}\left((\tilde{\gamma}_0 + \tilde{\gamma}_1)\tilde{\gamma}_5 + \zeta\tilde{\gamma}_{23}\right)} = \rho e^{\frac{1}{2}\zeta\tilde{\gamma}_{23}}(1 + A) \,, \tag{7.24}$$

where $\rho \in Spin_c(6)$ and A is a nilpotent generator, $A^2 = 0$. Decompose $\Delta^-_{\mathrm{Spin}(4,2)} \otimes \Delta^-_{\mathrm{Spin}(6)} = V_+ \oplus V_-$ as $\tilde{\gamma}_{01}\epsilon_\pm = \pm\epsilon_\pm$. Then the invariance condition can be written as

$$\rho \, e^{\frac{1}{2}\zeta\tilde{\gamma}_{23}}\epsilon_- = \epsilon_- \,,$$
$$\rho \, e^{\frac{1}{2}\zeta\tilde{\gamma}_{23}}(\epsilon_+ + \tilde{\gamma}_{05}\epsilon_-) = \epsilon_+ \,. \tag{7.25}$$

To preserve 30 supersymmetries, the first condition must be satisfied on an at least seven-dimensional complex subspace W_- of V_-. In turn this implies that an at least seven-dimensional subspace W_+ of V_+ is also invariant. Thus if $\epsilon_+ \in W_+$, one concludes that $\tilde{\gamma}_{50}\epsilon_- = 0$, and since $\tilde{\gamma}_{50}$ is invertible, $\epsilon_- = 0$, i.e. the spinors in W_- are not invariant. Therefore such quotients cannot preserve 30 supersymmetries. In fact one can show that $\hat{\alpha}$ preserves at most 16 supersymmetries.

The proof for cases 15 and 17 is similar. In addition, 3 and 5 are special cases. In all these cases, $N = 30$ quotients can be excluded.

7.2.3 Case 7 and 19

Let us begin with case 19. The lifted element can be written as

$$\hat{\alpha} = \rho e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}e^{A+\zeta B} \,, \tag{7.26}$$

where $\rho \in Spin_c(6)$ and

$$A = \frac{1}{2}(\tilde{\gamma}_5 + \tilde{\gamma}_1)(\tilde{\gamma}_0 + \tilde{\gamma}_2) \,,$$
$$B = \frac{1}{2}(\tilde{\gamma}_{02} - \tilde{\gamma}_{51}) \,. \tag{7.27}$$

It is clear that the element generated by $\tilde{\gamma}_{34}$ commutes with all the other and

$$AB = BA = 0, \quad A^2 = 0, \quad B^2 = P_-, \quad B^3 = B \,, \tag{7.28}$$

where $P_\pm = \frac{1}{2}(1 \pm \tilde{\gamma}_{0251})$. Using these, one finds that

$$e^{A+\zeta B} = (1 + A)[P_+ + \cosh\zeta P_- + \sinh\zeta B] \,. \tag{7.29}$$

Decomposing $\Delta^-_{\mathrm{Spin}(4,2)} \otimes \Delta^-_{\mathrm{Spin}(6)} = V_{++} \oplus V_{+-} \oplus V_{-+} \oplus V_{--}$ according to the commuting projections constructed from $\tilde{\gamma}_{51}$ and $\tilde{\gamma}_{02}$, one finds that the invariance equation can be written as

$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}(\epsilon_{++} - 2\tilde{\gamma}_{05}\epsilon_{--}) = \epsilon_{++} \,,$$
$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}[\cosh\zeta\epsilon_{+-} + \sinh\zeta\epsilon_{+-}] = \epsilon_{+-} \,,$$
$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}[\cosh\zeta\epsilon_{-+} - \sinh\zeta\epsilon_{-+}] = \epsilon_{-+} \,,$$
$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}\epsilon_{--} = \epsilon_{--} \,. \tag{7.30}$$

To obtain backgrounds with 30 supersymmetries, the last equation should have at least three complex independent solutions ϵ_{--}. This means that there must exist angles θ, ψ and φ such that $\rho\, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}} = 1$ for some selection of σ signs. Substituting this into the first equation, since the kernel of $\tilde{\gamma}_{05}$ is trivial, consistency requires that $\epsilon_{--} = 0$. Thus such solutions break more than 30 supersymmetries. In addition, case 7 can be treated in a similar way.

7.2.4 Cases 6, 8, 20 and 21

The lifted element in case 20 can be written as

$$\hat{\alpha} = \rho e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}} e^{A+\zeta B} , \tag{7.31}$$

where $\rho \in Spin_c(6)$ and

$$A = \frac{1}{2}(\tilde{\gamma}_5 + \tilde{\gamma}_1)(\tilde{\gamma}_0 + \tilde{\gamma}_2) ,$$
$$B = \frac{1}{2}(\tilde{\gamma}_{05} + \tilde{\gamma}_{12}) . \tag{7.32}$$

Next observe that

$$A^2 = 0 , \quad AB = BA , \quad B^2 = -P_- , \quad B^3 = -B , \quad P_\pm = \frac{1}{2}(1 \pm \tilde{\gamma}_{0512}) . \tag{7.33}$$

Using these, it is straightforward to show that

$$e^{A+\zeta B} = (1 + A)[P_+ + \cos\zeta P_- + \sin\zeta B] . \tag{7.34}$$

The rest of the analysis to exclude quotients which preserve 30 supersymmetries is similar to that of case 19 above. In addition, cases 6, 8 and 21 can be treated in a similar way. All these cases do not give quotients with 30 supersymmetries.

7.2.5 Cases 9 and 22

The lifted element in case 22 is

$$\hat{\alpha} = \rho\, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}} e^{\zeta A+\lambda B} , \tag{7.35}$$

where

$$A = \frac{1}{2}(\tilde{\gamma}_{05} - \tilde{\gamma}_{12}) , \quad B = \frac{1}{2}(\tilde{\gamma}_{02} - \tilde{\gamma}_{51}) . \tag{7.36}$$

Observe that

$$AB = BA = 0 , \quad A^2 = -P_+ , \quad A^3 = -A , \quad B^2 = P_- , \quad B^3 = B , \tag{7.37}$$

where $P_\pm = \frac{1}{2}(1 \pm \tilde{\gamma}_{0512})$. Using these we find that

$$\begin{aligned}
e^{\zeta A+\lambda B} &= (P_- + \cos\zeta\, P_+ + \sin\zeta\, A)(P_+ + \cosh\lambda\, P_- + \sinh\lambda\, B) \\
&= \cosh\lambda\, P_- + \cos\zeta\, P_+ + \sinh\lambda\, B + \sin\zeta\, A .
\end{aligned} \tag{7.38}$$

Decompose $\Delta^-_{\text{Spin}(4,2)} \otimes \Delta^-_{\text{Spin}(6)} = V_+ \oplus V_-$ using the projectors constructed from $\tilde{\gamma}_{0512}$. Observing that $B\epsilon_+ = A\epsilon_- = 0$, one can write the invariance equation as

$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}}[\cos\zeta \, \epsilon_+ + \sin\zeta \tilde{\gamma}_{05}\epsilon_+ + \cosh\lambda\epsilon_- + \sinh\lambda\tilde{\gamma}_{02}\epsilon_-] = \epsilon_+ + \epsilon_- \, . \tag{7.39}$$

Since $\tilde{\gamma}_{05}$ and $\tilde{\gamma}_{02}$ commute with the projectors constructed from $\tilde{\gamma}_{0512}$, one can rewrite the invariance equations as

$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}} \, e^{\zeta\tilde{\gamma}_{05}}\epsilon_+ = \epsilon_+ \, ,$$
$$\rho \, e^{\frac{1}{2}\varphi\tilde{\gamma}_{34}} \, e^{\lambda\tilde{\gamma}_{02}}\epsilon_- = \epsilon_- \, . \tag{7.40}$$

The above invariance conditions can be simplified somewhat by observing that the $\text{Spin}(4,2)$ chirality condition on the spinors together with the projections constructed from $\tilde{\gamma}_{0512}$ imply that $\tilde{\gamma}_{34}\epsilon_\pm = \mp i\epsilon_\pm$. To preserve 30 supersymmetries either V_+ or V_- must have a seven-dimensional invariant subspace. Using a similar argument to the one we have presented in cases 24 and 25, one can easily show that if V_+ has a seven-dimensional invariant subspace, then all of V_+ is invariant, and similarly for V_-. Therefore there are no such quotients with 30 supersymmetries. Case 9 can be analyzed in a similar way.

7.2.6 Case 13

The lifted element in this case is

$$\hat{\alpha} = \rho \, e^A \, , \tag{7.41}$$

where

$$A = \frac{1}{2}(\tilde{\gamma}_{05} + \tilde{\gamma}_{01} + \tilde{\gamma}_{03} - \tilde{\gamma}_{52} - \tilde{\gamma}_{12} - \tilde{\gamma}_{23}) \, . \tag{7.42}$$

Observe that

$$A^2 = -\tilde{\gamma}_{023}(\tilde{\gamma}_1 + \tilde{\gamma}_5) \, , \quad A^3 = \frac{1}{2}\tilde{\gamma}_{12}(1 + \tilde{\gamma}_{02})(1 + \tilde{\gamma}_{15}) \, . \tag{7.43}$$

Decomposing the spinors using the projectors constructed by $\tilde{\gamma}_{15}$ and $\tilde{\gamma}_{02}$, one finds that the invariance equation can be decomposed as

$$\rho \, \epsilon_{++} = \epsilon_{++} \, ,$$
$$\rho \, (\epsilon_{+-} + \tilde{\gamma}_{03}\epsilon_{++}) = \epsilon_{+-} \, ,$$
$$\rho \, (\epsilon_{-+} + 2\tilde{\gamma}_{01}\epsilon_{+-} + \tilde{\gamma}_{13}\epsilon_{++}) = \epsilon_{-+} \, ,$$
$$\rho \, (\epsilon_{--} + \tilde{\gamma}_{03}\epsilon_{-+} - \tilde{\gamma}_{13}\epsilon_{+-} + \frac{1}{3}\tilde{\gamma}_{12}\epsilon_{++}) = \epsilon_{--} \, . \tag{7.44}$$

It is straightforward from these to argue that there are no so such quotients which preserve 30 supersymmetries.

7.2.7 Cases 18 and 23

The lifted element for case 18 is

$$\hat{\alpha} = \rho \, e^{\zeta A + B} \,, \tag{7.45}$$

where

$$A = \frac{1}{2}(\mp \tilde{\gamma}_{05} + \tilde{\gamma}_{12} + \tilde{\gamma}_{34}) \,, \quad B = \frac{1}{2}(\tilde{\gamma}_{03} - \tilde{\gamma}_{13} \pm \tilde{\gamma}_{54} - \tilde{\gamma}_{24}) \,. \tag{7.46}$$

Next observe that

$$[A, B] = 0 \,, \quad B^3 = 0 \,. \tag{7.47}$$

Using these and without loss of generality choosing one of the signs in (7.46), one finds that the equation for invariance can be written as

$$\rho \, e^{\zeta A}[\epsilon_{++} + \epsilon_{--} + \tilde{\gamma}_{03}\epsilon_{+-} + \tilde{\gamma}_{54}\epsilon_{-+} + \tilde{\gamma}_{0543}\epsilon_{++}] = \epsilon_{--} + \epsilon_{++} \,,$$
$$\rho \, e^{\zeta A}[\epsilon_{+-} + \epsilon_{-+} + \tilde{\gamma}_{03}\epsilon_{++} + \tilde{\gamma}_{54}\epsilon_{++}] = \epsilon_{+-} + \epsilon_{-+} \,, \tag{7.48}$$

where we have decomposed $\Delta^-_{\mathrm{Spin}(4,2)} \otimes \Delta^-_{\mathrm{Spin}(6)} = V_{++} \oplus V_{-+} \oplus V_{+-} \oplus V_{--}$ with respect to the projectors constructed from $\tilde{\gamma}_{01}$ and $\tilde{\gamma}_{52}$, and use the property of A to commute with $\tilde{\gamma}_{0152}$. In addition, using the property of A to commute with the projectors $\frac{1}{4}(1 \pm \tilde{\gamma}_{01})(1 \pm \tilde{\gamma}_{52})$, with the signs correlated, the first equation in (7.48) can be decomposed further as

$$\rho \, e^{\zeta A} \epsilon_{++} = \epsilon_{++} \,,$$
$$\rho \, e^{\zeta A}[\epsilon_{--} + \tilde{\gamma}_{03}\epsilon_{+-} + \tilde{\gamma}_{54}\epsilon_{-+} + \tilde{\gamma}_{0543}\epsilon_{++}] = \epsilon_{--} \,. \tag{7.49}$$

To preserve 30 supersymmetries, the first equation above has to have at least three solutions. On these solutions, one can show that $\rho e^{\frac{i}{2}\zeta} = 1$. On the three dimensional eigenspace in V_{--} of $\rho \, e^{\zeta A}$ with the same eigenvalues consistency requires that

$$\tilde{\gamma}_{03}\epsilon_{+-} + \tilde{\gamma}_{54}\epsilon_{-+} + \tilde{\gamma}_{0543}\epsilon_{++} = 0 \,. \tag{7.50}$$

This condition can be solved to express at least three complex components of ϵ in terms of the remaining 13 components. Thus there are not 15 independent complex solutions to the invariance condition, and so such quotients cannot preserve 30 supersymmetries. The case 23 can be treated in a similar way.

7.3 Discrete quotients of plane wave

The isometry superalgebra[2] of the maximally supersymmetric plane wave [14] is

$$\begin{aligned}
&[e_-, e_i] = e_i^* \,, && [e_-, e_i^*] = -4\lambda^2 e_i \,, && [e_i^*, e_j] = -4\lambda^2 \delta_{ij} e_+ \,, \\
&[M_{ij}, e_k] = -\delta_{ik}e_j + \delta_{jk}e_i \,, && [M_{ij}, e_k^*] = -\delta_{ik}e_j^* + \delta_{jk}e_i^* \,, && i, j = 1, 2, 3, 4 \text{ and } 6, 7, 8, 9 \\
&[e_+, Q] = 0 \,, && [e_-, Q] = i\lambda(I + J)Q \,, && \\
&[e_i, Q] = -i\lambda I\Gamma_i\Gamma_+ Q \,, && [e_i^*, Q] = -2\lambda^2 I\Gamma_i\Gamma_+ Q \,, && i = 1, 2, 3, 4 \\
&[e_i, Q] = -i\lambda J\Gamma_i\Gamma_+ Q \,, && [e_i^*, Q] = -2\lambda^2 J\Gamma_i\Gamma_+ Q \,, && i = 6, 7, 8, 9 \\
&[M_{ij}, Q] = \frac{1}{2}\Gamma_{ij}Q \,, && I = \Gamma_{1234} \,, && J = \Gamma_{6789} \,, \tag{7.51}
\end{aligned}$$

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[2]We have not included the anti-commutator of the odd generators Q because it is not used in the analysis.

where λ is a real parameter. It can be read off from (7.51) that the isometry algebra of the maximally supersymmetric plane wave is $\mathfrak{so}(4) \oplus \mathfrak{so}(4) \oplus_s \mathfrak{t}$, where $\mathfrak{t} = \mathfrak{so}(2) \oplus_s \mathfrak{h}_{17}$ and $\mathfrak{h}_{17}$ is a Heisenberg algebra. The most general element of the isometry Lie algebra is

$$X = u^+ e_+ + v^- e_- + v^i e_i + w^i e_i^* + \frac{1}{2}\theta^{ij} M_{ij} \,, \tag{7.52}$$

where the indices i and i,j are restricted as in (7.51). Up to a conjugation, X can be brought to either

$$X = u^+ e_+ + v^- e_- + \sum_{n=0,n\neq 2}^{4} w^{2n+1} e_{2n+1}^* + \theta^1 M_{12} + \theta^2 M_{34} + \theta^3 M_{67} + \theta^4 M_{89} \tag{7.53}$$

if $v^- \neq 0$, or

$$X = u^+ e_+ + \sum_{i=1,i\neq 5}^{9} v^i e_i + \sum_{n=0,n\neq 2}^{4} w^{2n+1} e_{2n+1}^* + \theta^1 M_{12} + \theta^2 M_{34} + \theta^3 M_{67} + \theta^4 M_{89} \tag{7.54}$$

if $v^- = 0$. The action of the isometries on the Killing spinors can be read off from the commutators of the generators of the isometries with those of super-translations. In particular a lifted element is

$$\hat{\alpha} = e^{A+B} \,, \tag{7.55}$$

where

$$A = iv^-\lambda(I + J) + \frac{1}{2}(\theta^1 \Gamma_{12} + \theta^2 \Gamma_{34} + \theta^3 \Gamma_{67} + \theta^4 \Gamma_{89}) + i\psi \,,$$
$$B = -\lambda\left[I \sum_{i=1}^{4} \Gamma_i (iv^i + 2\lambda w^i) + J \sum_{i=6}^{9} \Gamma_i (iv^i + 2\lambda w^i) \right] \Gamma_+ \,. \tag{7.56}$$

The lifted generator $\hat{\alpha}$ has been partially adapted to the normal forms of X but the expression above will suffice for the analysis that follows. The Killing spinors are invariant along e_+ translations and so any identification along this direction preserves all supersymmetry. Writing $\epsilon = \epsilon_+ + \epsilon_-$ with $\Gamma_+ \epsilon_+ = 0$, we find that the invariance condition can be written as

$$e^A \epsilon_- = \epsilon_- \,,$$
$$e^A(\epsilon_+ + \Gamma_+ \beta \epsilon_-) = \epsilon_+ \,, \tag{7.57}$$

where β is a linear map that can be determined. Let us start by examining the first equation. The chirality of IIB spinors together with the lightcone projection implies that $(I + J)\epsilon_- = 0$. Therefore only the rotation part of e^A acts on ϵ_-. Thus one has

$$e^{\frac{i}{2}\sum_{i=1}^{4} \sigma_i \theta_i + i\psi} \epsilon_- = \epsilon_- \,, \qquad \sigma_1 \sigma_2 \sigma_3 \sigma_4 = -1 \,. \tag{7.58}$$

The restriction on the σ is due to the chirality condition on the spinors. There are 8 choices of signs giving rise to 8 independent conditions. $N = 30$ supersymmetry requires that at

least 7 conditions must hold. However one can show that if 7 conditions hold, then they imply the 8th. Moreover $\theta_i = 2\pi n_i$ and $\psi = n_0\pi$, where $n_0, n_i \in \mathbb{Z}$. These angles are associated with the identity rotation which lifts to the identity element, so in what follows we shall set $\theta_i = \psi = 0$. However observe that the invariance condition on ϵ_- does not restrict v^-.

Next let us turn to the second equation and consider the case $v^- = 0$. Then to preserve 30 supersymmetries, the kernel of β should have complex dimension 7. It turns out that

$$\beta\epsilon_- = \lambda[I \sum_{i=1,i\neq5}^{9} \Gamma_i(iv^i + 2\lambda w^i)]\epsilon_- . \tag{7.59}$$

So there is a non-trivial kernel iff

$$-v^2 + 4\lambda^2 w^2 - 4i\lambda v \cdot w = 0, \tag{7.60}$$

which in turn implies that $v \cdot w = 0$ and $v^2 = 4\lambda^2 w^2$. However in such a case the kernel has dimension 4 or 8. The latter occurs if $v = w = 0$. Thus there are no $N = 30$ quotients for $v^- = 0$.

Next let us consider the case where $v^- \neq 0$. In such a case the e_- generator acts non-trivially on ϵ_+. To continue observe that $\hat{\alpha}$ factorizes as

$$\hat{\alpha} = e^{iv^-\lambda I - \lambda I \sum_{i=1}^{4} \Gamma_i(iv^i + 2\lambda w^i)\Gamma_+} \; e^{iv^-\lambda J - \lambda J \sum_{i=6}^{9} \Gamma_i(iv^i + 2\lambda w^i)\Gamma_+} . \tag{7.61}$$

Using that I and $I\Gamma_i\Gamma_+$ anti-commute and the latter is nilpotent, and similarly for J and $J\Gamma_i\Gamma_+$, and after some computation, one finds that

$$e^{2i\lambda v^- I}\epsilon_+ + \Gamma_+ \frac{\sin(\lambda v^-)e^{i\lambda v^- I}}{\lambda v^-} \sum_{i=1,i\neq5}^{9} [i\lambda v^i + 2\lambda^2 w^i]I\Gamma_i\epsilon_- = \epsilon_+ . \tag{7.62}$$

Thus one has that

$$\beta = \frac{\sin(\lambda v^-)e^{i\lambda v^- I}}{\lambda v^-} \sum_{i=1,i\neq5}^{9} [i\lambda v^i + 2\lambda^2 w^i]I\Gamma_i . \tag{7.63}$$

As in the case with $v^- = 0$, we have to investigate the kernel of β. If $\lambda v^- = n\pi$, $n \in \mathbb{Z}-\{0\}$, then all supersymmetry is preserved. As it can be seen, it is remarkable that the Killing spinors in [14] are periodic in v^- with precisely this period. If $\lambda v^- \neq n\pi$, then β has a non-trivial kernel iff $v^2 = 4\lambda^2 w^2$ and $v \cdot w = 0$. As in the case with $v^- = 0$, one concludes that the kernel has dimension either 4 or 8. Thus such quotients do not preserve 30 supersymmetries.

8. Concluding remarks

We have shown that all $N > 28$ supersymmetric IIB backgrounds are maximally supersymmetric. The proof relies on the property that these backgrounds have vanishing one-form and three-form fluxes, $P = G = 0$, which arises as consequence of the homogeneity of

$N > 24$ backgrounds and the algebraic Killing spinor equation of IIB supergravity. In addition, the supercovariant curvature vanishes subject to the field equations and the Bianchi identities of the theory. Therefore all $N > 28$ supersymmetric IIB backgrounds are locally maximally supersymmetric. Finally, $28 < N < 32$ backgrounds cannot be constructed as discrete quotients of maximally supersymmetric ones.

It is natural to ask whether it is possible to extend the above results to other near maximal backgrounds with $N \leq 28$. This does not seem straightforward. In particular, it is known that there are plane wave backgrounds with 28 supersymmetries [25, 26]. Significantly, these backgrounds have non-vanishing three-form flux, $G \neq 0$. Thus apart from the maximally supersymmetric case, 7/8 is the highest fraction of supersymmetry that IIB backgrounds preserve.

The existence of backgrounds with 28 supersymmetries does not necessarily imply that there are supersymmetric backgrounds for all $N < 28$. Some more fractions of supersymmetry may be excluded as a conjecture in [15] indicates. Such cases will exhibit supersymmetry enhancement similar to that we have shown for backgrounds with $N > 28$. It would be of interest to classify all IIB backgrounds with 28 supersymmetries as the first near maximal case that has solutions which do not have maximal supersymmetry. This may be possible using the homogeneity of these backgrounds.

Our results can be extended to investigate nearly maximally supersymmetric IIA backgrounds. This is because of the similarities between the Killing spinor equations of IIA and IIB supergravities; in particular both have an algebraic Killing spinor equation. In fact, it appears that the nearly maximally supersymmetric solutions of IIA supergravity are more restricted than those of IIB. In particular, there is a unique maximally supersymmetric IIA solution, the Minkowski spacetime, and the $N = 31$ IIA backgrounds are maximally supersymmetric. The $N = 30$ IIA backgrounds can be investigated in a way similar to those of IIB by appropriately modifying the IIB complex linearity argument for the IIA dilatino Killing spinor equation and showing that the supercovariant curvature vanishes.

In eleven-dimensions, the investigation of nearly maximally supersymmetric backgrounds is more involved. This is because eleven-dimensional supergravity does not have an algebraic Killing spinor equation. So an extension of our results to eleven-dimensions depends crucially on the properties of the gravitino Killing spinor equation. Nevertheless, it would be of interest to see whether the results of [3] can be extended to backgrounds with less than 31 supersymmetries.

Acknowledgments

References

[1] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest, *$N = 31$ is not IIB*, *JHEP* **02** (2007) 044 [hep-th/0606049].

[2] I.A. Bandos, J.A. de Azcarraga and O. Varela, *On the absence of BPS preonic solutions in IIA and IIB supergravities*, *JHEP* **09** (2006) 009 [hep-th/0607060].

[3] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest, *$N = 31$, $D = 11$*, *JHEP* **02** (2007) 043 [hep-th/0610331].

[4] J. Figueroa-O'Farrill and S. Gadhia, *M-theory preons cannot arise by quotients*, *JHEP* **06** (2007) 043 [hep-th/0702055].

[5] I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo and J. Lukierski, *BPS states in M-theory and twistorial constituents*, *Phys. Rev. Lett.* **86** (2001) 4451 [hep-th/0101113];
I.A. Bandos, J.A. de Azcarraga, J.M. Izquierdo, M. Picón and O. Varela, *On BPS preons, generalized holonomies and $D = 11$ supergravities*, *Phys. Rev.* **D 69** (2004) 105010 [hep-th/0312266].

[6] J. Gillard, U. Gran and G. Papadopoulos, *The spinorial geometry of supersymmetric backgrounds*, *Class. and Quant. Grav.* **22** (2005) 1033 [hep-th/0410155].

[7] J. Figueroa-O'Farrill and J. Simon, *Supersymmetric Kaluza-Klein reductions of AdS backgrounds*, *Adv. Theor. Math. Phys.* **8** (2004) 217 [hep-th/0401206].

[8] J. Figueroa-O'Farrill, O. Madden, S.F. Ross and J. Simon, *Quotients of $AdS(p + 1) \times S^q$: causally well-behaved spaces and black holes*, *Phys. Rev.* **D 69** (2004) 124026 [hep-th/0402094].

[9] J. Grover, J.B. Gutowski and W.A. Sabra, *Maximally minimal preons in four dimensions*, *Class. and Quant. Grav.* **24** (2007) 3259 [hep-th/0610128]; *Vanishing preons in the fifth dimension*, *Class. and Quant. Grav.* **24** (2007) 417 [hep-th/0608187].

[10] J. Figueroa-O'Farrill, J. Gutowski and W. Sabra, *The return of the four- and five-dimensional preons*, *Class. and Quant. Grav.* **24** (2007) 4429 [arXiv:0705.2778].

[11] J. Figueroa-O'Farrill, E. Hackett-Jones and G. Moutsopoulos, *The Killing superalgebra of ten-dimensional supergravity backgrounds*, *Class. and Quant. Grav.* **24** (2007) 3291 [hep-th/0703192].

[12] J. Figueroa-O'Farrill and G. Papadopoulos, *Maximally supersymmetric solutions of ten- and eleven-dimensional supergravities*, *JHEP* **03** (2003) 048 [hep-th/0211089].

[13] J.H. Schwarz, *Covariant field equations of chiral $N = 2$ $D = 10$ supergravity*, *Nucl. Phys.* **B 226** (1983) 269.

[14] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A new maximally supersymmetric background of IIB superstring theory*, *JHEP* **01** (2002) 047 [hep-th/0110242].

[15] M.J. Duff, *M-theory on manifolds of G_2 holonomy: the first twenty years*, hep-th/0201062.

[16] J.H. Schwarz and P.C. West, *Symmetries and transformations of chiral $N = 2$ $D = 10$ supergravity*, *Phys. Lett.* **B 126** (1983) 301.

[17] P.S. Howe and P.C. West, *The complete $N = 2$, $D = 10$ supergravity*, *Nucl. Phys.* **B 238** (1984) 181.

[18] U. Gran, J. Gutowski and G. Papadopoulos, *The spinorial geometry of supersymmetric IIB backgrounds*, *Class. and Quant. Grav.* **22** (2005) 2453 [`hep-th/0501177`].

[19] U. Gran, J. Gutowski, G. Papadopoulos and D. Roest, *Systematics of IIB spinorial geometry*, *Class. and Quant. Grav.* **23** (2006) 1617 [`hep-th/0507087`].

[20] G. Papadopoulos and D. Tsimpis, *The holonomy of IIB supercovariant connection*, *Class. and Quant. Grav.* **20** (2003) L253 [`hep-th/0307127`].

[21] P.S. Howe, J.M. Izquierdo, G. Papadopoulos and P.K. Townsend, *New supergravities with central charges and Killing spinors in* $(2+1)$-*dimensions*, *Nucl. Phys.* **B 467** (1996) 183 [`hep-th/9505032`].

[22] B.R. Greene, A.D. Shapere, C. Vafa and S.-T. Yau, *Stringy cosmic strings and noncompact Calabi-Yau manifolds*, *Nucl. Phys.* **B 337** (1990) 1.

[23] G.W. Gibbons, M.B. Green and M.J. Perry, *Instantons and seven-branes in type IIB superstring theory*, *Phys. Lett.* **B 370** (1996) 37 [`hep-th/9511080`].

[24] E. Bergshoeff, U. Gran and D. Roest, *Type IIB seven-brane solutions from nine-dimensional domain walls*, *Class. and Quant. Grav.* **19** (2002) 4207 [`hep-th/0203202`].

[25] I. Bena and R. Roiban, *Supergravity pp-wave solutions with* 28 *and* 24 *supercharges*, *Phys. Rev.* **D 67** (2003) 125014 [`hep-th/0206195`].

[26] J. Michelson, *A pp-wave with* 26 *supercharges*, *Class. and Quant. Grav.* **19** (2002) 5935 [`hep-th/0206204`].